

The thermal expansion coefficient α

$$\alpha = -\frac{1}{\rho} \left. \frac{\partial \rho}{\partial \theta} \right|_{s, p}$$

$$\rho(s, \theta, p) = \frac{\rho(s, \theta, 0) k(s, \theta, p)}{k(s, \theta, p) - p}$$

$$\begin{aligned} \alpha &= -\frac{(k-p) \frac{\partial}{\partial \theta} (\rho_0 k) - \rho_0 k \frac{\partial}{\partial \theta} (k-p)}{\rho (k-p) (k-p)} \\ &= -\frac{(k-p) \left(\rho_0 \frac{\partial k}{\partial \theta} + k \frac{\partial \rho_0}{\partial \theta} \right) - \rho_0 k \frac{\partial k}{\partial \theta}}{\rho (k-p) (k-p)} \\ &= -\frac{\frac{\partial \rho_0}{\partial \theta} (k^2 - pk) - p \rho_0 \frac{\partial k}{\partial \theta}}{\rho (k-p) (k-p)} \end{aligned}$$

$$\alpha = -\frac{k(k-p) \frac{\partial \rho_0}{\partial \theta} - p \rho_0 \frac{\partial k}{\partial \theta}}{\rho (k-p) (k-p)}$$

$$\alpha = -\frac{k(k+0.1z) \frac{\partial \rho_0}{\partial \theta} + 0.1z \rho_0 \frac{\partial k}{\partial \theta}}{\rho (k+0.1z)^2}$$

The saline contraction coefficient β

$$\beta = \frac{1}{\rho} \left. \frac{\partial \rho}{\partial s} \right|_{\theta, p}$$

$$\begin{aligned} \beta &= \frac{(k-p) \frac{\partial}{\partial s} (\rho_0 k) - \rho_0 k \frac{\partial}{\partial s} (k-p)}{\rho (k-p) (k-p)} \\ &= \frac{(k-p) \left(\rho_0 \frac{\partial k}{\partial s} + k \frac{\partial \rho_0}{\partial s} \right) - \rho_0 k \frac{\partial k}{\partial s}}{\rho (k-p) (k-p)} \\ &= \frac{\frac{\partial \rho_0}{\partial s} (k^2 - pk) - p \rho_0 \frac{\partial k}{\partial s}}{\rho (k-p) (k-p)} \end{aligned}$$

$$\beta = \frac{k(k-p) \frac{\partial \rho_0}{\partial s} - p \rho_0 \frac{\partial k}{\partial s}}{\rho (k-p) (k-p)}$$

$$\beta = \frac{k(k+0.1z) \frac{\partial \rho_0}{\partial s} + 0.1z \rho_0 \frac{\partial k}{\partial s}}{\rho (k+0.1z)^2}$$