

Isopycnic Tensor (Sasha Implementation)

$$R(T) = -\rho \cdot \vec{F}(T)$$

The components of the diffusive flux are:

$$-F^x = \frac{1}{2} \left(\frac{\partial T}{\partial x} - \frac{\partial T}{\partial z} \frac{\partial \rho}{\partial x} / \frac{\partial \rho}{\partial z} \right)$$

$$-F^y = \frac{1}{2} \left(\frac{\partial T}{\partial y} - \frac{\partial T}{\partial z} \frac{\partial \rho}{\partial y} / \frac{\partial \rho}{\partial z} \right)$$

$$-F^z = \frac{1}{2} \left[-\frac{\partial T}{\partial x} \frac{\partial \rho}{\partial x} / \frac{\partial \rho}{\partial z} - \frac{\partial T}{\partial y} \frac{\partial \rho}{\partial y} / \frac{\partial \rho}{\partial z} + \frac{\partial T}{\partial z} \left(\frac{\partial \rho}{\partial x} / \frac{\partial \rho}{\partial z} \right)^2 + \frac{\partial T}{\partial z} \left(\frac{\partial \rho}{\partial y} / \frac{\partial \rho}{\partial z} \right)^2 \right]$$

If we use the following identities

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial \rho} \frac{\partial \rho}{\partial z}$$

or

$$\frac{\partial T}{\partial \rho} = \frac{\partial T}{\partial z} / \frac{\partial \rho}{\partial z}$$

the components of the flux becomes

$$-F^x = \frac{1}{2} \left(\frac{\partial T}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial T}{\partial \rho} \right)$$

$$-F^y = \frac{1}{2} \left(\frac{\partial T}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial T}{\partial \rho} \right)$$

$$-F^z = \frac{1}{2} \left[\frac{\partial \rho}{\partial x} \left(\frac{\partial \rho}{\partial x} \frac{\partial T}{\partial \rho} - \frac{\partial T}{\partial x} \right) + \frac{\partial \rho}{\partial y} \left(\frac{\partial \rho}{\partial y} \frac{\partial T}{\partial \rho} - \frac{\partial T}{\partial y} \right) \right] / \frac{\partial \rho}{\partial z}$$