Griffies et al. Isopycnic Mixing

The isopycnic diffusion operator can be written as

$$R(T) = -\nabla\cdot\vec{F}(T)$$

where $\vec{F}(T)$ are the diffusive fluxes

$$F^{m}(T) = -K^{mn}\partial_n T$$

and K^{mn} is the isopicnic diffusion tensor. The small slope approximation of this tensor is:

$$K^{small} = \nu_I \begin{bmatrix} 1 & 0 & S_x \\ 0 & 1 & S_y \\ S_x & S_y & \epsilon + S^2 \end{bmatrix} \qquad \epsilon = \frac{\nu_D}{\nu_I} \sim 10^{-7} \text{ to } 10^{-8}$$

and

$$S = -\nabla_z \rho \Big/ \frac{\partial \rho}{\partial z}$$

To guarantee diffusion along isoneutral surfaces and positive definiteness, derive from a discretized functional

$$\vec{G}(T) = -\frac{1}{2}\int K\left(\frac{\nabla T \times \nabla \rho}{|\nabla\rho|^2}\right)dV = \frac{1}{2}\int(\nabla T\cdot\vec{F})\,dV$$

In the small limit approximation: $\rho_x, \rho_y \ll \rho_z$

$$\vec{G}(T) = -\frac{1}{2}\int \nu_I\left[\left(\frac{\partial T}{\partial x} - \frac{\partial T}{\partial z}S_x\right)^2 + \left(\frac{\partial T}{\partial y} - \frac{\partial T}{\partial z}S_y\right)^2\right]dV$$

$$-\frac{1}{2}\int \nu_D\left(\frac{\partial T}{\partial z}\right)^2 dV$$