The isopicnic diffusion tensor also can be written as

$$K = \frac{\nu_I}{(1+S^2)} \begin{pmatrix} 1+S_x^2+\epsilon S_x^2 & (\epsilon-1)S_xS_y & (1-\epsilon)S_x \\ (\epsilon-1)S_xS_y & 1+S_y^2+\epsilon S_y^2 & (1-\epsilon)S_y \\ (\epsilon-1)S_x & (\epsilon-1)S_y & \epsilon+S^2 \end{pmatrix}$$

In an orthonormal isopycnal frame which separates the effects of anisotropy between the along and across isopycnal directions

$$K = \nu_I \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{\nu_I(\epsilon-1)}{\rho_x^2+\rho_y^2+\rho_z^2} \begin{pmatrix} \rho_x^2 & \rho_x\rho_y & \rho_x\rho_z \\ \rho_x\rho_y & \rho_y^2 & \rho_y\rho_z \\ \rho_x\rho_z & \rho_y\rho_z & \rho_z^2 \end{pmatrix}$$

<u>Small angle Approximation</u>

If the isopycnic slope are small, $S \ll 1$, the isopycnic diffusion tensor reduces to

$$K_{small} = \nu_I \begin{pmatrix} 1 & 0 & (\epsilon-1)\rho_x/\rho_z \\ 0 & 1 & (\epsilon-1)\rho_y/\rho_z \\ (\epsilon-1)\rho_x/\rho_x & (\epsilon-1)\rho_y/\rho_z & \epsilon+S^2 \end{pmatrix} = \nu_1 \begin{pmatrix} 1 & 0 & (\epsilon-1)S_x \\ 0 & 1 & (\epsilon-1)S_y \\ (1-\epsilon)S_x & (1-\epsilon)S_y & \epsilon+S^2 \end{pmatrix}$$

In an orthonormal isopycnic frame the small angle approximation of the tensor becomes

$$K_{small} = \begin{pmatrix} \nu_I & 0 & 0 \\ 0 & \nu_I(1+S^2) & 0 \\ 0 & 0 & \nu_D \end{pmatrix} + \frac{\nu_D S^2}{1+S^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & S \\ 0 & S & 1 \end{pmatrix}$$