Isopycnic Tensor

$$A: \begin{pmatrix} A_{xx} & A_{yx} & A_{zx} \\ A_{xy} & A_{yy} & A_{zy} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix}$$

The isopycnic diffusion operator, $R(T)$, can be written as

$$R(T) = - \nabla \cdot \vec{F}(T)$$

where $\vec{F}(T)$ are the diffusive fluxes

$$F^{m}(T) = - K^{mn} \partial_n T$$

where K^{mn} is the diffusion tensor

$$K = \frac{\nu_I}{(1+S^2)} \begin{pmatrix} 1 + \frac{\rho_x^2 + \epsilon \rho_x^2}{\rho_z^2} & (\epsilon - 1)\frac{\rho_x \rho_y}{\rho_z^2} & (\epsilon-1)\frac{\rho_x}{\rho_z} \\ (\epsilon-1)\frac{\rho_x \rho_y}{\rho_z^2} & 1 + \frac{\rho_y^2 + \epsilon \rho_y}{\rho_z^2} & (\epsilon-1)\frac{\rho_y}{\rho_z} \\ (\epsilon-1)\frac{\rho_x}{\rho_z} & (\epsilon-1)\frac{\rho_y}{\rho_z} & \epsilon + S^2 \end{pmatrix}$$

and

$$\vec{S} = \nabla_\rho z = - z_\rho \nabla_z \rho = (S_x, S_y, 0) = \left(-\frac{\rho_x}{\rho_z}, -\frac{\rho_y}{\rho_z}, 0\right)$$

$$\epsilon = \frac{\nu_D}{\nu_I} \sim 10^{-7}$$

ν_I = along isopycnal diffusion coefficient

ν_D = diapycnal diffusion coefficient

$\vec{S}$ = isopycnal slope vector