<u>OMEGA</u> vertical velocity equation:

The omega equation can be derived from 3D and 2D continuity equations:

$$\frac{\partial}{\partial t}\left(\frac{H_\theta}{mn}\right) + \frac{\partial}{\partial \xi}\left(\frac{H_\theta u}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v}{m}\right) + \frac{\partial}{\partial s}\left(\frac{H_\theta \Omega}{mn}\right) = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}\left(\frac{\zeta}{mn}\right) + \frac{\partial}{\partial \xi}\left(\frac{D\bar{u}}{n}\right) + \frac{\partial}{\partial \xi}\left(\frac{D\bar{v}}{m}\right) = 0 \qquad (2)$$

Subtracting (2) from (1) gives

$$\cancelto{0}{\frac{\partial}{\partial t}\left(\frac{H_\theta - \zeta}{mn}\right)} + \frac{\partial}{\partial \xi}\left(\frac{H_\theta u - D\bar{u}}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v - D\bar{v}}{m}\right) + \frac{\partial}{\partial s}\left(\frac{H_\theta \Omega}{mn}\right) = 0$$

$$H_\theta \equiv \frac{\partial z}{\partial s} = (\zeta + h_c) + (h - h_c)C_d \qquad \frac{\partial H_\theta}{\partial t} = \frac{\partial \zeta}{\partial t}$$

Then

$$\frac{\partial}{\partial s}\left(\frac{H_\theta \Omega}{mn}\right) = -\frac{\partial}{\partial \xi}\left(\frac{H_\theta u - D\bar{u}}{n}\right) - \frac{\partial}{\partial \eta}\left(\frac{H_\theta v - D\bar{v}}{m}\right)$$

Integrating

$$\cancelto{0}{\left.\frac{H_\theta \Omega}{mn}\right|_{s'=0}} - \left.\frac{H_\theta \Omega}{mn}\right|_{s'=s} = -\int_s^0 \left[\frac{\partial}{\partial \xi}\left(\frac{H_\theta u - D\bar{u}}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v - D\bar{v}}{m}\right)\right] ds$$

Then the vertical velocity is

$$\boxed{\frac{H_\theta \Omega}{mn} = \int_s^0 \left[\frac{\partial}{\partial \xi}\left(\frac{H_\theta u - D\bar{u}}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v - D\bar{v}}{m}\right)\right] ds} \qquad (3)$$