

let vertical velocity equation:

$$K_Q = l g \delta_Q$$

$$\delta_Q = k S_M$$

(7)

and using $\frac{\partial}{\partial z} = \frac{1}{H_0} \frac{\partial}{\partial s}$ and adding horizontal dissipation $\frac{\partial}{\partial t}$ gives

$$\frac{\partial}{\partial t} \left(\frac{H_0 q^2 l}{mn} \right) + \frac{\partial}{\partial z} \left(\frac{H_0 u q^2 l}{n} \right) + \frac{\partial}{\partial y} \left(\frac{H_0 v q^2 l}{m} \right) + \frac{\partial}{\partial s} \left(\frac{H_0 \omega q^2 l}{mn} \right) - \frac{H_0}{mn} \frac{1}{H_0} \frac{\partial}{\partial s} \left[\frac{K_Q}{H_0} \frac{\partial}{\partial s} (q^2 l) \right] = \frac{H_0}{mn} l E_1 (P_s + P_b) - \frac{H_0}{mn} \frac{q^3}{B_1} \sqrt{W} + \frac{H_0}{mn} \frac{\partial}{\partial t} q^2$$

or

$$\frac{\partial}{\partial t} \left(\frac{H_0 q^2 l}{mn} \right) + \frac{\partial}{\partial z} \left(\frac{H_0 u q^2 l}{n} \right) + \frac{\partial}{\partial y} \left(\frac{H_0 v q^2 l}{m} \right) + \frac{\partial}{\partial s} \left(\frac{H_0 \omega q^2 l}{mn} \right) - \frac{\partial}{\partial s} \left[\frac{K_Q}{mn H_0} \frac{\partial}{\partial s} (q^2 l) \right] = \frac{H_0}{mn} l E_1 (P_s + P_b) - \frac{H_0}{mn} \frac{q^3}{B_1} \sqrt{W} + \frac{H_0}{mn} \frac{\partial}{\partial t} q^2$$

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$$\frac{\partial}{\partial t} \left(\frac{H_0 q^2 l}{mn} \right) = \frac{H_0}{mn} \int \left[\frac{\partial}{\partial z} \left(\frac{H_0 u q^2 l}{n} \right) + \frac{\partial}{\partial y} \left(\frac{H_0 v q^2 l}{m} \right) \right] ds$$

then the vertical velocity is

$$\frac{H_0 \omega}{mn} = \int \left[\frac{\partial}{\partial z} \left(\frac{H_0 u q^2 l}{n} \right) + \frac{\partial}{\partial y} \left(\frac{H_0 v q^2 l}{m} \right) \right] ds$$