Turbulent Length Scale Equation - Level 2.5 closure

The governing equation for the turbulent energy times the scale (is)

$$\frac{D}{Dt}(q^2 l) - \frac{\partial}{\partial z}\left[q l S_l \frac{\partial}{\partial z}(q^2 l)\right] = l E_1 \left[P_s + P_b\right] - \frac{q^3}{B_1}\tilde{W} \qquad (1)$$

where $\tilde{W}$ is the wall proximity function

$$\tilde{W} = 1 + E_2\left(\frac{l}{\kappa L}\right)^2 \qquad (2)$$

$$L^{-1} = (\zeta - z)^{-1} + (H + z)^{-1}$$

or

$$L = \left[\frac{1}{\frac{1}{(\zeta - z)} + \frac{1}{(H+z)}}\right] \qquad (3)$$

and κ is the Von Karman constant, P_s is the shear production of turbulent energy, P_b is the buoyant production of turbulent energy.

$$P_s = K_M \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \qquad (4)$$

$$P_b = K_H \left[\frac{g}{\rho_o}\frac{\partial \rho}{\partial z}\right] = -K_H N^2 \qquad (5)$$

Using the material derivative in (1), after multiplying by $\left(\frac{H\theta}{mn}\right)$ yields

$$\frac{\partial}{\partial t}\left(\frac{H\theta q^2 l}{mn}\right) + \frac{\partial}{\partial \xi}\left(\frac{H\theta u q^2 l}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H\theta v q^2 l}{m}\right) + \frac{\partial}{\partial s}\left(\frac{H\theta \Omega q^2 l}{mn}\right) \qquad (6)$$

$$- \frac{H\theta}{mn}\frac{\partial}{\partial z}\left(q l S_l \frac{\partial}{\partial z}(q^2 l)\right) = \frac{H\theta}{mn} l E_1 \left[P_s + P_b\right] - \frac{H\theta}{mn}\frac{q^3}{B_1}\tilde{W}$$