b) Biharmonic

$$\frac{H_\theta}{mn} Q_q = \beta(q^2) = A_{HB}\left[\frac{\partial}{\partial \xi}\left(F_q^{\xi}\right) + \frac{\partial}{\partial \eta}\left(F_q^{\eta}\right)\right] \qquad (27)$$

$$\frac{m^4}{s}\ \frac{m}{s^2} = \frac{m^5}{s^3}$$

where

$$F_q^{\xi} = \frac{m}{n} H_\theta \frac{\partial}{\partial \xi}\left[\frac{mn}{H_\theta} \mathcal{L}(q^2)\right] \qquad (28)$$

$$F_q^{\eta} = \frac{n}{m} H_\theta \frac{\partial}{\partial \eta}\left[\frac{mn}{H_\theta} \mathcal{L}(q^2)\right] \qquad (29)$$

$$nm \quad \frac{1}{m}\frac{1}{m}\frac{1}{m}\frac{m^5}{s^2} = \frac{m}{s^2}$$

Tridiagonal Set-up

$$\frac{\partial}{\partial t}(H_\theta q^2) = mnR_q + P_s + P_b - \varepsilon_d + \frac{\partial}{\partial s}\left[\frac{K_Q}{H_\theta}\frac{\partial}{\partial s}(q^2)\right]$$

Using Crank-Nicolson scheme in the vertical viscosity term, such that

$$\phi = (1-\lambda)\phi^{n} + \lambda \phi^{n+1} \qquad \lambda = 0.5$$

gives

$$\frac{\partial}{\partial t}(H_\theta q^2) = mnR_q + P_s + P_b - \varepsilon_d + (1-\lambda)\frac{\partial}{\partial s}\left(\frac{K_Q}{H_\theta^n}\frac{\partial q^{2^n}}{\partial s}\right) + \lambda\frac{\partial}{\partial s}\left(\frac{K_Q}{H_\theta^{n+1}}\frac{\partial q^{2^{n+1}}}{\partial s}\right)$$

$$\frac{H_\theta^{n+1} q^{2^{n+1}} - H_\theta^{n} q^{2^{n}}}{\Delta t} = mnR_q + P_s + P_b - \varepsilon_d + \frac{(1-\lambda)}{\Delta s^2}\left[\frac{K_{Q\,k+1}}{H^n_{\theta_{k+1}}}\left(q^{2^n}_{k+1} - q^{2^n}_{k}\right) - \frac{K_{Qk}}{H^n_{\theta_k}}\left(q^{2^n}_{k} - q^{2^n}_{k-1}\right)\right]$$

$$+ \frac{\lambda}{\Delta s^2}\left[\frac{K_{Q\,k+1}}{H^{n+1}_{\theta_{k+1}}}\left(q^{2^{n+1}}_{k+1} - q^{2^{n+1}}_{k}\right) - \frac{K_{Qk}}{H^{n+1}_{\theta_k}}\left(q^{2^{n+1}}_{k} - q^{2^{n+1}}_{k-1}\right)\right]$$

$$H^{n+1}_{\theta_k} q^{2^{n+1}}_{k} - \frac{\lambda \Delta t}{\Delta s^2}\left[\frac{K_{Q\,k+1}}{H^{n+1}_{\theta_{k+1}}}\left(q^{2^{n+1}}_{k+1} - q^{2^{n+1}}_{k}\right) - \frac{K_{Qk}}{H^{n+1}_{\theta_k}}\left(q^{2^{n+1}}_{k} - q^{2^{n+1}}_{k-1}\right)\right] = H^n_{\theta_k} q^{2^n} + \Delta t\, mnR_{q_k}$$

$$+ \Delta t\left(P_s + P_b - \varepsilon_d\right) + \frac{\Delta t(1-\lambda)}{\Delta s^2}\left[\frac{K_{Q\,k+1}}{H^n_{\theta_{k+1}}}\left(q^{2^n}_{k+1} - q^{2^n}_{k}\right) - \frac{K_{Qk}}{H^n_{\theta_k}}\left(q^{2^n}_{k} - q^{2^n}_{k-1}\right)\right]$$