Therefore, (4.7) becomes

$$\frac{\partial}{\partial t}\left(\frac{D\overline{u}}{mn}\right)+\frac{\partial}{\partial \xi}\left(\frac{D\overline{uu}}{n}\right)+\frac{\partial}{\partial \eta}\left(\frac{D\overline{vu}}{m}\right)-\frac{Df}{mn}\overline{v}-\left[\overline{vv}\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right)-\overline{uv}\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right]D=$$
$$-\frac{D}{n}\frac{\partial}{\partial \xi}\left(\overline{p_T}\right)+\frac{D}{mn}\left(\overline{\mathcal{D}_u}+\overline{F_u}\right)+\frac{1}{mn}\left(\tau_s^{\xi}-\tau_b^{\xi}\right) \qquad (5.3)$$

Similarly, η-component is

$$\frac{\partial}{\partial t}\left(\frac{D\overline{v}}{mn}\right)+\frac{\partial}{\partial \xi}\left(\frac{D\overline{uv}}{n}\right)+\frac{\partial}{\partial \eta}\left(\frac{D\overline{vv}}{m}\right)+\frac{Df}{mn}\overline{u}+\left[\overline{vu}\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right)-\overline{uu}\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right]D=$$
$$-\frac{D}{m}\frac{\partial}{\partial \eta}\left(\overline{p_T}\right)+\frac{D}{mn}\left(\overline{\mathcal{D}_v}+\overline{F_u}\right)+\frac{1}{mn}\left(\tau_s^{\eta}-\tau_b^{\eta}\right) \qquad (5.4)$$

(b) Vertically integrating continuity Equation

The vertically integrated continuity equation is obtain by integrating (4.5) and using (5.1), (5.2), and (2.5)

Notice that,

$$\int_{-1}^{0}\frac{\partial}{\partial t}\left(\frac{H\varrho}{mn}\right)=\frac{1}{mn}\frac{\partial}{\partial t}\int_{-1}^{0}\frac{\partial z}{\partial s}\,ds$$
$$=\frac{1}{mn}\frac{\partial}{\partial t}\left[z(s=0)-z(s=-1)\right]$$
$$z(s=0)=\zeta,\quad z(s=-1)=-h(\xi,\eta)$$
$$=\frac{1}{mn}\frac{\partial}{\partial t}\left(\zeta+h\right)=\frac{1}{mn}\frac{\partial}{\partial t}(D)$$

$$\int_{-1}^{0}\frac{\partial}{\partial t}\left(\frac{H\varrho}{mn}\right)ds=\frac{\partial}{\partial t}\left(\frac{\zeta}{mn}\right) \quad \text{or} \quad \int_{-1}^{0}\frac{\partial}{\partial t}\left(\frac{H\varrho}{mn}\right)ds=\frac{\partial}{\partial t}\left(\frac{D}{mn}\right) \qquad (5.5)$$