Therefore, the total pressure gradient in (x,y,z,t) coordinates is

$$\boxed{\nabla_z p_T = g\nabla_z \zeta + \nabla_z \phi} \qquad (4.12)$$

The dynamic pressure in (x,y,s,t) coordinates is

$$\boxed{\phi(x,y,s,t) = \frac{g}{\rho_0}\int_s^0 \rho(x,y,s,t)\frac{\partial z}{\partial s}\,ds} \qquad (4.13)$$

Evaluation of $\nabla_z \phi$

using identity (2.9) $\nabla_z \phi$ becomes

$$\nabla_z \phi = \nabla_s \phi - \frac{\partial \phi}{\partial z}\nabla_s z$$

since $\frac{\partial \phi}{\partial z} = -\frac{g}{\rho_0}\rho$ and using (4.13) yields

$$\boxed{\nabla_z \phi = \frac{g}{\rho_0}\nabla_s\left[\int_s^0 \rho\frac{\partial z}{\partial s}\,ds\right] + \frac{g}{\rho_0}\rho\nabla_s z} \qquad (4.14)$$

Since the gradient and integral are both in s, the differentiation and integration can be interchanged (Leibnitz's rule).

$$\nabla_z \phi = \frac{g}{\rho_0}\int_s^0 \nabla_s\left(\rho\frac{\partial z}{\partial s}\right)ds + \frac{g}{\rho_0}\rho\nabla_s z$$