

Multiplying (4.4) by $\frac{H_0}{mn}$ gives

$$\frac{H_0}{mn} \frac{\partial v}{\partial t} + \frac{H_0 k}{n} \frac{\partial v}{\partial z} + \frac{H_0 v}{m} \frac{\partial v}{\partial \eta} + \frac{H_0}{mn} \frac{\partial \partial v}{\partial s} + \frac{H_0 f}{mn} u + \left[v \frac{\partial}{\partial z} \left(\frac{1}{n} \right) - u \frac{\partial}{\partial \eta} \left(\frac{1}{m} \right) \right] H_0 u = -\frac{H_0}{m} \frac{\partial p}{\partial \eta} + \frac{H_0}{mn} F$$

Using chain rule

$$\frac{H_0}{mn} \frac{\partial v}{\partial t} = \frac{\partial}{\partial t} \left(\frac{H_0 v}{mn} \right) - v \frac{\partial}{\partial t} \left(\frac{H_0}{mn} \right)$$

$$\frac{H_0 k}{n} \frac{\partial v}{\partial z} = \frac{\partial}{\partial z} \left(\frac{H_0 k v}{n} \right) - v \frac{\partial}{\partial z} \left(\frac{H_0 k}{n} \right)$$

$$\frac{H_0 v}{m} \frac{\partial v}{\partial \eta} = \frac{\partial}{\partial \eta} \left(\frac{H_0 v^2}{m} \right) - v \frac{\partial}{\partial \eta} \left(\frac{H_0 v}{m} \right)$$

$$\frac{H_0}{mn} \frac{\partial \partial v}{\partial s} = \frac{\partial}{\partial s} \left(\frac{H_0 \partial v}{mn} \right) - v \frac{\partial}{\partial s} \left(\frac{H_0}{mn} \right)$$

rearranging

$$\frac{\partial}{\partial t} \left(\frac{H_0 v}{mn} \right) + \frac{\partial}{\partial z} \left(\frac{H_0 k v}{n} \right) + \frac{\partial}{\partial \eta} \left(\frac{H_0 v^2}{m} \right) + \frac{\partial}{\partial s} \left(\frac{H_0 \partial v}{mn} \right) + \frac{f}{mn} u + \left[v \frac{\partial}{\partial z} \left(\frac{1}{n} \right) - u \frac{\partial}{\partial \eta} \left(\frac{1}{m} \right) \right] H_0 u = -\frac{H_0}{m} \frac{\partial p}{\partial \eta} + \frac{H_0}{mn} F + v \left[\frac{\partial}{\partial t} \left(\frac{H_0}{mn} \right) + \frac{\partial}{\partial z} \left(\frac{H_0 k}{n} \right) + \frac{\partial}{\partial \eta} \left(\frac{H_0 v}{m} \right) + \frac{\partial}{\partial s} \left(\frac{H_0}{mn} \right) \right]$$

Therefore

$$\frac{\partial}{\partial t} \left(\frac{H_0 v}{mn} \right) + \frac{\partial}{\partial z} \left(\frac{H_0 k v}{n} \right) + \frac{\partial}{\partial \eta} \left(\frac{H_0 v^2}{m} \right) + \frac{\partial}{\partial s} \left(\frac{H_0 \partial v}{mn} \right) + \frac{H_0 f}{mn} u + \left[v \frac{\partial}{\partial z} \left(\frac{1}{n} \right) - u \frac{\partial}{\partial \eta} \left(\frac{1}{m} \right) \right] H_0 u = -\frac{H_0}{m} \frac{\partial p}{\partial \eta} + \frac{H_0}{mn} F$$

where

$$\frac{H_0}{mn} F = \frac{H_0}{mn} (D_r + F_v) + \frac{\partial}{\partial s} \left(\frac{K_m}{H_0 mn} \frac{\partial u}{\partial s} \right)$$

(4.8)