© Flux Form of Momentum Equations

Multiplying (4.3) by $\frac{H_\theta}{mn}$ gives

$$\frac{H_\theta}{mn}\frac{\partial u}{\partial t} + \frac{H_\theta u}{n}\frac{\partial u}{\partial \xi} + \frac{H_\theta v}{m}\frac{\partial v}{\partial \eta} + \frac{H_\theta}{mn}\Omega\frac{\partial u}{\partial s} - \frac{H_\theta f}{mn}v - \left[v\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right) - u\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right]v H_\theta =$$

$$- \frac{H_\theta}{n}\frac{\partial P_T}{\partial \xi} + \frac{H_\theta}{mn}F_\xi$$

Using chaine rule

$$\frac{H_\theta}{mn}\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{H_\theta u}{mn}\right) - u\frac{\partial}{\partial t}\left(\frac{H_\theta}{mn}\right)$$

$$\frac{H_\theta}{n}u\frac{\partial u}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{H_\theta u u}{n}\right) - u\frac{\partial}{\partial \xi}\left(\frac{H_\theta u}{n}\right)$$

$$\frac{H_\theta}{m}v\frac{\partial u}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{H_\theta v u}{m}\right) - u\frac{\partial}{\partial \eta}\left(\frac{H_\theta v}{m}\right)$$

$$\frac{H_\theta}{mn}\Omega\frac{\partial u}{\partial s} = \frac{\partial}{\partial s}\left(\frac{H_\theta \Omega u}{mn}\right) - u\frac{\partial}{\partial s}\left(\frac{H_\theta \Omega}{mn}\right)$$

rearranging

$$\frac{\partial}{\partial t}\left(\frac{H_\theta u}{mn}\right) + \frac{\partial}{\partial \xi}\left(\frac{H_\theta u u}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v u}{m}\right) + \frac{\partial}{\partial s}\left(\frac{H_\theta \Omega u}{mn}\right) - \frac{H_\theta f}{mn}v - \left[v\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right) - u\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right]v H_\theta =$$

$$- \frac{H_\theta}{n}\frac{\partial P_T}{\partial \xi} + \frac{H_\theta}{mn}F_\xi + u\underbrace{\left[\frac{\partial}{\partial t}\left(\frac{H_\theta}{mn}\right) + \frac{\partial}{\partial \xi}\left(\frac{H_\theta u}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v}{m}\right) + \frac{\partial}{\partial s}\left(\frac{H_\theta \Omega}{mn}\right)\right]}_{0}$$

Therefore

$$\boxed{\frac{\partial}{\partial t}\left(\frac{H_\theta u}{mn}\right) + \frac{\partial}{\partial \xi}\left(\frac{H_\theta u u}{n}\right) + \frac{\partial}{\partial \eta}\left(\frac{H_\theta v u}{m}\right) + \frac{\partial}{\partial s}\left(\frac{H_\theta \Omega u}{mn}\right) - \frac{H_\theta f}{mn}v - \left[v\frac{\partial}{\partial \xi}\left(\frac{1}{n}\right) - u\frac{\partial}{\partial \eta}\left(\frac{1}{m}\right)\right]H_\theta v = - \frac{H_\theta}{n}\frac{\partial P_T}{\partial \xi} + \frac{H_\theta}{mn}F_\xi}$$

$m^3 \frac{m}{s}$

Where

$$\frac{H_\theta}{mn}F_\xi = \frac{H_\theta}{mn}\left(\mathcal{D}_u + F_u\right) + \frac{\partial}{\partial s}\left(\frac{K_m}{H_\theta mn}\frac{\partial u}{\partial s}\right) \qquad (4.7)$$