3.- Orthogonal Curvilinear Coordinates

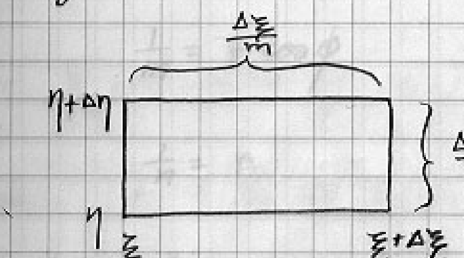
Let the orthogonal curvilinear coordinates ξ and η, and let the actual distances corresponding to $d\xi$ and $d\eta$ be $(ds)_\xi$ and $(ds)_\eta$, respectively.

Define the metric factors m, n such that

$$(ds)_\xi = \frac{1}{m}\, d\xi \qquad m: \text{ meters}^{-1} \tag{3.1}$$

$$(ds)_\eta = \frac{1}{n}\, d\eta \qquad n: \text{ meters}^{-1} \tag{3.2}$$

Rectangular area element:



$$\frac{\partial A}{\partial \xi} = \frac{\partial A}{\partial x}\frac{\partial x}{\partial \xi} = \frac{1}{m}\frac{\partial A}{\partial x}$$

$$\frac{\partial A}{\partial x} = m\frac{\partial A}{\partial \xi}$$

$$\frac{\partial A}{\partial \eta} = \frac{\partial A}{\partial y}\frac{\partial y}{\partial \eta} = \frac{1}{n}\frac{\partial A}{\partial y}$$

$$\frac{\partial A}{\partial y} = n\frac{\partial A}{\partial \eta}$$

ⓐ Flux form of divergence:

$$\nabla_s \cdot \vec{V} = \lim_{\substack{\Delta\xi \to 0 \\ \Delta\eta \to 0}} \frac{\delta_\xi\left[u\left(\frac{\Delta\eta}{n}\right)\right] + \delta_\eta\left[v\left(\frac{\Delta\xi}{m}\right)\right]}{\left(\frac{\Delta\xi}{m}\right)\left(\frac{\Delta\eta}{n}\right)}$$

$$\boxed{\nabla_s \cdot \vec{V} = mn\left[\frac{\partial}{\partial\xi}\left(\frac{u}{n}\right) + \frac{\partial}{\partial\eta}\left(\frac{v}{m}\right)\right]} \tag{3.3}$$

ⓑ Flux form of vertical Curl

$$\boxed{\hat{k}\cdot\nabla_s \times \vec{V} = mn\left[\frac{\partial}{\partial\xi}\left(\frac{v}{n}\right) - \frac{\partial}{\partial\eta}\left(\frac{u}{m}\right)\right]} \tag{3.4}$$