(f) Flux Form of Material Derivative

Multiplying the advective derivative (2.7) by H_θ and using continuity equation (2.12)

$$H_\theta \frac{\partial A}{\partial t} + H_\theta (\vec{V} \cdot \nabla_3 A) + H_\theta \Omega \frac{\partial A}{\partial s} = 0$$

but
$$H_\theta \frac{\partial A}{\partial t} = \frac{\partial (H_\theta A)}{\partial t} - A \frac{\partial H_\theta}{\partial t}$$
$$H_\theta (\vec{V} \cdot \nabla_3 A) = \nabla_3 \cdot (H_\theta \vec{V} A) - A\left[\nabla_3 \cdot (H_\theta \vec{V})\right]$$
$$H_\theta \Omega \frac{\partial A}{\partial s} = \frac{\partial}{\partial s}(H_\theta \Omega A) - A \frac{\partial}{\partial s}(H_\theta \Omega)$$

Therefore,

$$\frac{\partial (H_\theta A)}{\partial t} + \nabla_3 \cdot (H_\theta \vec{V} A) + \frac{\partial}{\partial s}(H_\theta \Omega A) = A \underbrace{\left[\frac{\partial H_\theta}{\partial t} + \nabla_3 \cdot (H_\theta \vec{V}) + \frac{\partial}{\partial s}(H_\theta \Omega)\right]}_{0}$$

$$\boxed{H_\theta \frac{DA}{Dt} = \frac{\partial (H_\theta A)}{\partial t} + \nabla_3 \cdot (H_\theta \vec{V} A) + \frac{\partial}{\partial s}(H_\theta \Omega A) = 0} \quad \text{flux form} \qquad (2.13)$$

where A is any fluid property (scalar or vector).