Antidiffusion Velocities

$$\tilde{u} = \frac{1}{2}\left(u\,\Delta\xi - \Delta t\, u^2\right)\frac{1}{T}\frac{\partial T}{\partial \xi} - \frac{1}{2}\Delta t\, u\left(v\frac{1}{T}\frac{\partial T}{\partial \eta} + w\frac{1}{T}\frac{\partial T}{\partial z}\right)$$

$$\tilde{v} = \frac{1}{2}\left(v\,\Delta\eta - \Delta t\, v^2\right)\frac{1}{T}\frac{\partial T}{\partial \eta} - \frac{1}{2}\Delta t\, v\left(u\frac{1}{T}\frac{\partial T}{\partial \xi} + w\frac{1}{T}\frac{\partial T}{\partial z}\right)$$

$$\tilde{W} = \frac{1}{2}\left(W\Delta z - \Delta t\, W^2\right)\frac{1}{T}\frac{\partial T}{\partial z} - \frac{1}{2}\Delta t\, W\left(u\frac{1}{T}\frac{\partial T}{\partial \xi} + v\frac{1}{T}\frac{\partial T}{\partial \eta}\right)$$

$$\frac{L}{t}\,L \qquad \cancel{t}\,\frac{L^2}{t^2}\;\frac{1}{L} \qquad \cancel{t}\,\frac{L}{\cancel{t}}\;\frac{L}{T}\,\frac{1}{L} \qquad \frac{L}{T}\,\frac{1}{L}$$

The term $\frac{1}{T}\frac{\partial T}{\partial z}$ also can be written as

$$\frac{1}{T}\frac{\partial T}{\partial z} = \frac{1}{T H_\theta \Delta s}\frac{\partial T}{\partial s}$$

in s-coordinates, but is not necessary when coding. The code will be better structured in z-coordinates

W is assumed here as in terms of true velocity units, so

$$\omega_a = W_{model}\,(mn) \qquad W_{model} = \frac{\Omega H_\theta}{mn} \qquad \frac{\Omega H_\theta}{\frac{1}{s}\ m}$$

Limits on transport fluxes

$$\tilde{u} = \min(1, \beta\!\downarrow_{i-1}, \beta\!\uparrow_i)\max(0, \tilde{u}_i) + \min(1, \beta\!\uparrow_{i-1}, \beta\!\downarrow_i)\min(0, \tilde{u}_i)$$

$$\tilde{v} = \min(1, \beta\!\downarrow_{j-1}, \beta\!\uparrow_j)\max(0, \tilde{v}_j) + \min(1, \beta\!\uparrow_{j-1}, \beta\!\downarrow_j)\min(0, \tilde{v}_j)$$

$$\tilde{W} = \min(1, \beta\!\downarrow_k, \beta\!\uparrow_{k+1})\max(0, \tilde{W}_k) + \min(1, \beta\!\uparrow_k, \beta\!\downarrow_{k+1})\min(0, \tilde{W}_k)$$

where $\beta\!\downarrow$ and $\beta\!\uparrow$ are the β-ratios used to preserve monotonicity.