January 30, 1996

Smolarkiewicz Revisited

Tracer advection equation:

$$\frac{\partial}{\partial t}\left(\frac{H_z T}{mn}\right) = -\frac{\partial}{\partial \xi}\left(\frac{H_z u T}{n}\right) - \frac{\partial}{\partial \eta}\left(\frac{H_z v T}{m}\right) - \frac{\partial}{\partial s}\left(\frac{H_z \Omega T}{mn}\right)$$

F^{ξ} F^{η} F^{s}

Numerical Scheme

(1) Define generic "upwind" advective flux function

$$AFLUX(T_1, T_2, V) = ((V + abs(V))T_1 + (V - abs(V))T_2)$$

(2) Compute advective fluxes:

$$F^{\xi} = AFLUX\left(T(i-1,j,k),\ T(i,j,k),\ \frac{u H_z}{n}\right)$$ $i=1,L$; $j=1,M_m$; $k=1,N$

→ @ U-RHO points

$$F^{\eta} = AFLUX\left(T(i,j-1,k),\ T(i,j,k),\ \frac{v H_z}{m}\right)$$ $i=1,L_m$; $j=1,M$; $k=1,N$

→ @ V-RHO points

$$F^{s} = AFLUX\left(T(i,j,k),\ T(i,j,k+1),\ \frac{w H_z}{mn}\right)$$

→ @ RHO-W points

NOTE: Before using vertical velocity, scale it back to velocity units, that is, recall that $W = \frac{\Omega H_0}{mn}$ in the model

$$w_a = W_{model}(mn) \qquad W = \frac{\Omega H_0}{\frac{1}{s}\ m}$$

(3) Compute advected tracer

$$T^{n+1}_{i,j,k} = T^{n}_{i,j,k} - \frac{\Delta t\, mn}{H_z}\left(\frac{\partial F^{\xi}}{\partial \xi} + \frac{\partial F^{\eta}}{\partial \eta} + \frac{\partial F^{s}}{\partial s}\right)$$