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Alternative

$$f^{n+1} - f^{n} = (\alpha f^{n} + \beta f^{n-1} + \gamma f^{n-2}) \quad (1)$$

$$\frac{f^{n+1} - f^{n}}{\Delta t} = f' + \left(\frac{\Delta t}{2}\right) f'' + \left(\frac{\Delta t^2}{6}\right) f''' + \left(\frac{\Delta t^3}{24}\right) f'''' + \cdots \quad (2)$$

$$\alpha: \quad F f^{n} = f' \quad (3)$$

$$\beta: \quad F f^{n-1} = f' - (\Delta t) f'' + \left(\frac{\Delta t^2}{2}\right) f''' - \left(\frac{\Delta t^3}{6}\right) f'''' + \cdots \quad (4)$$

$$\gamma: \quad F f^{n-2} = f' - (2\Delta t) f'' + (2\Delta t^2) f''' - \left(\frac{8\Delta t^3}{6}\right) f'''' + \cdots \quad (5)$$

Solving for coefficients

$$\alpha + \beta + \gamma = 1 \quad (6)$$

$$-\beta - 2\gamma = 1/2 \quad (7)$$

$$\frac{1}{2}\beta + 2\gamma = \frac{1}{6} \quad (8)$$

$$\beta = -\frac{1}{2} - 2\gamma$$

$$\gamma = \frac{1}{2}\left(\frac{1}{6} - \frac{1}{2}\beta\right) = \frac{1}{12} - \frac{1}{4}\beta$$

$$\beta = -\frac{1}{2} - 2\left(\frac{1}{12} - \frac{1}{4}\right)\beta$$

$$\beta = -\frac{1}{2} - \frac{2}{12}\beta + \frac{2}{4}\beta$$

$$\beta\left(1 + \frac{2}{12} - \frac{2}{4}\right) = -\frac{1}{2}$$

$$\frac{8}{12}\beta = -\frac{1}{2}$$

$$\boxed{\beta = -\frac{16}{12}}$$

$$\gamma = \frac{1}{12} - \frac{1}{4}\left(-\frac{16}{12}\right)$$

$$= \frac{1}{12} + \frac{16}{48}$$

$$= \frac{20}{48}$$

$$\boxed{\gamma = \frac{5}{12}}$$

$$\alpha = 1 - \beta - \gamma$$

$$\alpha = 1 - \left(-\frac{16}{12}\right) - \frac{5}{12}$$

$$\boxed{\alpha = \frac{23}{12}}$$