

Third-Order Adams-Bashforth

The Taylor series expansion for F^{n+1} in time is

$$F^{n+1} = F^n + \frac{\partial F}{\partial t} \Big|_n \Delta t + \frac{1}{2!} \frac{\partial^2 F}{\partial t^2} \Big|_n \Delta t^2 + \frac{1}{3!} \frac{\partial^3 F}{\partial t^3} \Big|_n \Delta t^3 + O(\Delta t^4)$$

Evaluating derivatives by one-sided finite differences in time gives:

$$\frac{\partial^2 F}{\partial t^2} \Big|_n = \frac{\partial}{\partial t} \left(\frac{\partial F}{\partial t} \Big|_n \right) = \frac{\partial F}{\partial t} \Big|_n - \frac{\partial F}{\partial t} \Big|_{n-1}$$

$$\begin{aligned} \frac{\partial^3 F}{\partial t^3} \Big|_n &= \frac{\partial}{\partial t} \left(\frac{\partial^2 F}{\partial t^2} \Big|_n \right) = \frac{\partial^2 F}{\partial t^2} \Big|_n - \frac{\partial^2 F}{\partial t^2} \Big|_{n-1} = \frac{\partial F}{\partial t} \Big|_n - \frac{\partial F}{\partial t} \Big|_{n-1} - \frac{\partial F}{\partial t} \Big|_{n-1} + \frac{\partial F}{\partial t} \Big|_{n-2} \\ &= \frac{\partial F}{\partial t} \Big|_n - 2 \frac{\partial F}{\partial t} \Big|_{n-1} + \frac{\partial F}{\partial t} \Big|_{n-2} \end{aligned}$$

The right-hand-side terms of the model are:

$$n+1 \quad R_4$$

$$n \quad R_3$$

$$n-1 \quad R_2$$

$$n-2 \quad R_1$$

$$\frac{\partial F}{\partial t} \Big|_n = R_3$$

$$\frac{\partial^2 F}{\partial t^2} \Big|_n = \frac{R_3 - R_2}{\Delta t^2}$$

$$\frac{\partial^3 F}{\partial t^3} \Big|_n = \frac{R_3 - 2R_2 + R_1}{\Delta t^3}$$

Evaluating Taylor series gives

$$F^{n+1} = F^n + R_3 \Delta t + \frac{1}{2} \left(\frac{R_3 - R_2}{\Delta t^2} \right) \Delta t^2 + \frac{1}{6} \left(\frac{R_3 - 2R_2 + R_1}{\Delta t^3} \right) \Delta t^3 + \dots$$

Grouping right-hand-side terms gives:

$$\left(1 + \frac{1}{2} + \frac{1}{6}\right) R_3 + \left(-\frac{1}{2} - \frac{1}{3}\right) R_2 + \frac{1}{6} R_1$$

$$\left(\frac{12}{12} + \frac{6}{12} + \frac{2}{12}\right) R_3 + \left(-\frac{6}{12} - \frac{4}{12}\right) R_2 + \frac{2}{12} R_1$$

$$\left(\frac{20}{12}\right) R_3 + \left(-\frac{10}{12}\right) R_2 + \left(\frac{2}{12}\right) R_1$$

Paper/scrums 2.1

$$\frac{23}{12}$$

$$-\frac{16}{12}$$

$$\frac{5}{12}$$

Wrong derivation!