Jan 15, 1995

## SCRUM Tridiagonal System

$$\frac{\partial(H_\theta u)}{\partial t} = mnR_u + \frac{\partial}{\partial s}\left(\frac{K_M}{H_\theta}\frac{\partial u}{\partial s}\right) \qquad (1) \qquad \begin{cases} K_M \frac{\partial u}{\partial z} = \tau_s^x \\ K_M \frac{\partial u}{\partial z} = \tau_b^x \end{cases}$$

$$\frac{\partial(H_\theta v)}{\partial t} = mnR_v + \frac{\partial}{\partial s}\left(\frac{K_M}{H_\theta}\frac{\partial v}{\partial s}\right) \qquad (2) \qquad \begin{cases} K_M \frac{\partial v}{\partial z} = \tau_s^y \\ K_M \frac{\partial v}{\partial z} = \tau_b^y \end{cases}$$

$$\frac{\partial(H_\theta T)}{\partial t} = mnR_T + \frac{\partial}{\partial s}\left(\frac{K_H}{H_\theta}\frac{\partial T}{\partial s}\right) \qquad (3)$$

$$+ \frac{\partial}{\partial s}($$

Using Crank-Nicolson scheme in $s$ in the vertical viscosity/diffusion term, such that

$$\phi = (1-\lambda)\phi^n + \lambda\phi^{n+1} \qquad \lambda = 0.5$$

The discrete form of (1) becomes

$$\frac{\partial(H_\theta u)}{\partial t} = mnR_u + (1-\lambda)\frac{\partial}{\partial s}\left(\frac{K_H}{H_\theta^n}\frac{\partial u^n}{\partial s}\right) + \lambda\frac{\partial}{\partial s}\left(\frac{K_H}{H_\theta^{n+1}}\frac{\partial u^{n+1}}{\partial s}\right)$$

or

$$\frac{H_\theta^{n+1}u^{n+1} - H_\theta^n u^n}{\Delta t} = mnR_u + \frac{(1-\lambda)}{\Delta s^2}\left[\frac{K_{M_k}}{H_{\theta_k}^n}\left(u_{k+1}^n - u_k^n\right) - \frac{K_{M_{k-1}}}{H_{\theta_{k-1}}^n}\left(u_k^n - u_{k-1}^n\right)\right]$$

$$+ \frac{\lambda}{\Delta s^2}\left[\frac{K_{M_k}}{H_{\theta_k}^{n+1}}\left(u_{k+1}^{n+1} - u_k^{n+1}\right) - \frac{K_{M_{k-1}}}{H_{\theta_{k-1}}^{n+1}}\left(u_k^{n+1} - u_{k-1}^{n+1}\right)\right]$$

or

$$H_\theta^{n+1}u_k^{n+1} - \frac{\lambda\Delta t}{\Delta s^2}\left[\frac{K_{M_k}}{H_{\theta_k}^{n+1}}\left(u_{k+1}^{n+1} - u_k^{n+1}\right) - \frac{K_{M_{k-1}}}{H_{\theta_{k-1}}^{n+1}}\left(u_k^{n+1} - u_{k-1}^{n+1}\right)\right] = H_\theta^n u^n + \Delta t\, mnR_u +$$

$$\frac{\Delta t\,(1-\lambda)}{\Delta s^2}\left[\frac{K_{M_k}}{H_{\theta_k}^n}\left(u_{k+1}^n - u_k^n\right) - \frac{K_{M_{k-1}}}{H_{\theta_{k-1}}^n}\left(u_k^n - u_{k-1}^n\right)\right]$$