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Tridiagonal Solver

(Richtmeyer and Morton, 1967)

Commonly known as Thomas algorithm. Also its known as the Double-Sweep solution Method. It works for any PDE in which an implicit scheme can be arranged in the general form

$$A_k \phi_{k+1}^{n+1} + B_k \phi_k^{n+1} + C_k \phi_{k-1}^{n+1} = D_k \qquad (1)$$

To solve (1) for all k, we introduce two additional independent variables E_k and F_k such that

$$\phi_{k+1}^{n+1} = E_k \phi_k^{n+1} + F_k \qquad (2)$$

so that a linear relationship is assumed to exist between ϕ_{k+1} and ϕ_k at the next time level. Substituting (2) into (1) gives

$$A_k\left(E_k \phi_k^{n+1} + F_k\right) + B_k \phi_k^{n+1} + C_k \phi_{k-1}^{n+1} = D_k$$

$$A_k E_k \phi_k^{n+1} + A_k F_k + B_k \phi_k^{n+1} + C_k \phi_{k-1}^{n+1} = D_k$$

or

$$\left(A_k E_k + B_k\right) \phi_k^{n+1} + C_k \phi_{k-1}^{n+1} = D_k - A_k F_k$$

dividing through by $(A_k E_k + B_k)$ and rearranging gives

$$\phi_k^{n+1} = \left[\frac{-C_k}{A_k E_k + B_k}\right] \phi_{k-1}^{n+1} + \left[\frac{D_k - A_k F_k}{A_k E_k + B_k}\right] \qquad (3)$$

Equation (3) is comparable to (2) but indexed downward. That is, (3) could be written as

$$\phi_k^{n+1} = E_{k-1} \phi_{k-1}^{n+1} + F_{k-1} \qquad (4)$$

where

$$E_{k-1} = -\frac{C_k}{A_k E_k + B_k} \qquad (5)$$

$$F_{k-1} = \frac{D_k - A_k F_k}{A_k E_k + B_k} \qquad (6)$$