A ROMS state variable, ϕ, can be represented in terms of its time mean, $\bar{\phi}$, plus a set of N-tidal harmonics of frequency, ω_k.

$$\phi(t) = \bar{\phi} + \sum_{k=1}^{N} A_k \, sin\, \omega_k t + \sum_{k=1}^{N} B_k \, cos\, \omega_k t$$

The unknown tidal amplitude A_k, and B_k and unknown state ϕ coefficients are evaluated by minimizing the least-squares error function defined by:

$$\varepsilon^2 = \frac{1}{T} \int_{t_1}^{t_2} \left[\phi - \left(\bar{\phi} + \sum_{k=1}^{N} (A_k \, sin\, \omega_k t) + \sum_{k=1}^{N} (B_k \, cos\, \omega_k t) \right) \right]^2 dt$$

In discrete space:

$$\varepsilon^2 = \frac{1}{M} \sum_{i=1}^{M} \left[\phi_i - \left(\bar{\phi} + \sum_{k=1}^{N} (A_k \, sin\, \omega_k t_i) + \sum_{k=1}^{N} (B_k \, cos\, \omega_k t_i) \right) \right]^2$$

Minimization subject to the additional constraints $\frac{\partial \varepsilon^2}{\partial \phi} = 0$, $\frac{\partial \varepsilon^2}{\partial A_k} = 0$, $\frac{\partial \varepsilon^2}{\partial B_k} = 0$ result in a linear set of equations:

$$\sum_{i=1}^{M} \left[-2\phi_i + 2\bar{\phi} + 2 \sum_{k=1}^{N} (A_k \, sin\, \omega_k t_i) + 2 \sum_{k=1}^{N} (B_k \, sin\, \omega_k t_i) \right] = 0$$

$$\sum_{i=1}^{M} \left[-2\phi_i \, sin\, \omega_k t_i + 2\bar{\phi} \, sin\, \omega_k t_i + 2 \sum_{p=1}^{N} (A_p \, sin\, \omega_p t_i \, sin\, \omega_k t_i) + 2 \sum_{p=1}^{N} (B_p \, cos\, \omega_p t_i \, sin\, \omega_k t_i) \right] = 0$$

$$\sum_{i=1}^{M} \left[-2\phi_i \, cos\, \omega_k t_i + 2\bar{\phi} \, cos\, \omega_k t_i + 2 \sum_{p=1}^{N} (A_p \, sin\, \omega_p t_i \, cos\, \omega_k t_i) + 2 \sum_{p=1}^{N} (B_p \, cos\, \omega_p t_i \, cos\, \omega_k t_i) \right] = 0$$

in matrix form (N harmonics). Note: all instances of $\sum$ are actually $\sum_{i=1}^{M}$ where M is the number of time-steps in the time-averaging window.

$$
\underset{A}{\begin{bmatrix}
M & \sum sin\,\omega_1 t_i & \sum sin\,\omega_2 t_i & \cdots & \sum cos\,\omega_1 t_i & \cdots \\
\sum sin\,\omega_1 t_i & \sum sin^2\,\omega_1 t_i & \sum sin\,\omega_2 t_i\,sin\,\omega_1 t_i & \cdots & \sum cos\,\omega_1 t_i\,sin\,\omega_1 t_i & \cdots \\
\sum sin\,\omega_2 t_i & \sum sin\,\omega_1 t_i\,sin\,\omega_2 t_i & \sum sin^2\,\omega_2 t_i & \cdots & \sum cos\,\omega_1 t_i\,sin\,\omega_2 t_i & \cdots \\
\vdots & \vdots & \cdots & \cdots & \cdots & \cdots \\
\sum sin\,\omega_7 t_i & \sum sin\,\omega_1 t_i\,sin\,\omega_7 t_i & \sum sin\,\omega_2 t_i\,sin\,\omega_7 t_i & \cdots & \sum cos\,\omega_1 t_i\,sin\,\omega_7 t_i & \cdots \\
\sum sin\,\omega_1 t_i & \sum sin\,\omega_1 t_i\,cos\,\omega_1 t_i & \sum sin\,\omega_2 t_i\,cos\,\omega_1 t_i & \cdots & \sum cos^2\,\omega_1 t_i & \cdots \\
\sum cos\,\omega_2 t_i & \sum sin\,\omega_1 t_i\,cos\,\omega_2 t_i & \sum sin\,\omega_2 t_i\,cos\,\omega_2 t_i & \cdots & \sum cos\,\omega_1 t_i\,cos\,\omega_2 t_i & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\sum cos\,\omega_7 t_i & \sum sin\,\omega_1 t_i\,cos\,\omega_7 t_i & \sum sin\,\omega_2 t_2\,cos\,\omega_7 t_i & \cdots & \sum cos\,\omega_1 t_i\,cos\,\omega_7 t_i & \cdots
\end{bmatrix}}
\underset{x}{\begin{bmatrix}
\bar{\phi} \\ A_1 \\ A_2 \\ \vdots \\ A_7 \\ B_1 \\ B_2 \\ \vdots \\ B_7
\end{bmatrix}}
=
\underset{b}{\begin{bmatrix}
\sum \phi_i \\
\sum \phi_i\,sin\,\omega_1 ti \\
\sum \phi_i\,sin\,\omega_2 ti \\
\vdots \\
\sum \phi_i\,sin\,\omega_7 ti \\
\sum \phi_i\,cos\,\omega_1 ti \\
\sum \phi_i\,cos\,\omega_2 ti \\
\vdots \\
\sum \phi_i\,cos\,\omega_7 ti
\end{bmatrix}}
$$