Roms-Agrif Two-Way nesting algorithms:
Latest Developments

Laurent Debreu

INRIA, Laboratoire Jean Kuntzmann, Grenoble

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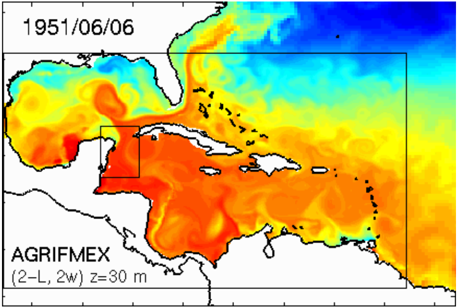
Following Debreu and Blayo, 2008: "Two-Way embedding algorithms: a review", *Ocean Dynamics*, In press

Outline

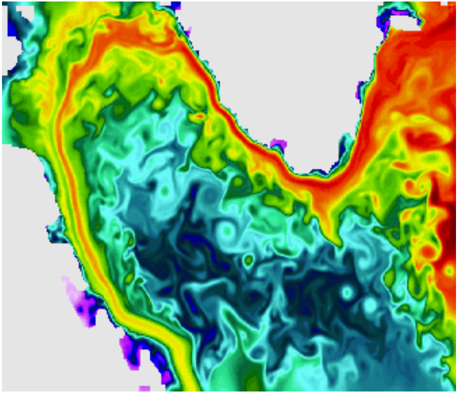
Outline

Examples of two way nesting applications

OPA Model

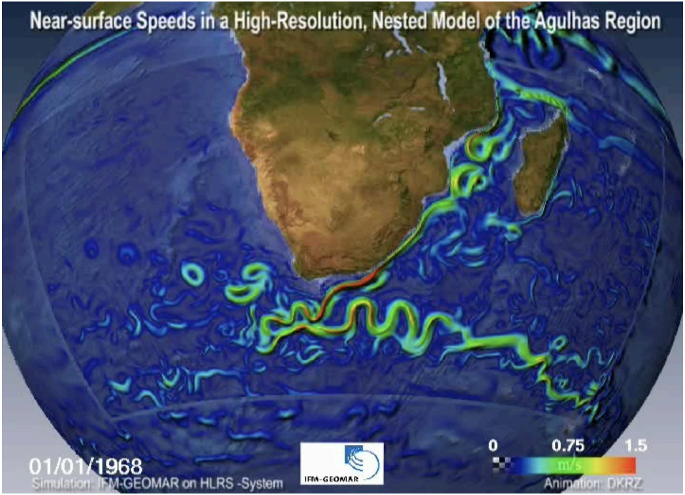


Jouanno et al, *Ocean Modelling*, 2008



Chanut et al, *JPO*, 2008

Examples of two way nesting applications
OPA Model

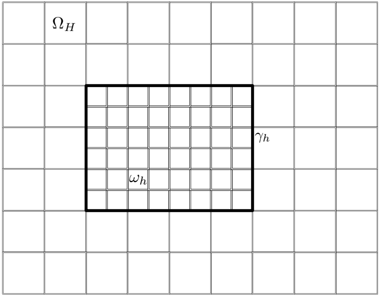


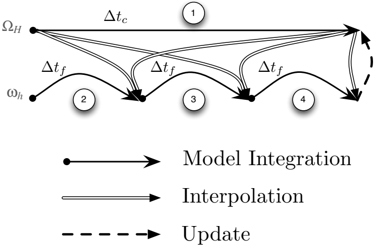
Biastoch et al, Nature, 2008

Outline

Outline

The grid hierarchy and its time integration





P: interpolation
R: restriction

1. $q_c^{n+1} = \mathcal{L}_c(q_c^n)$
2. For $m = 1 \ldots \rho_t$ do

$$q_f^{n+\frac{m}{\rho_t}} = \mathcal{L}_f\left(q_f^{n+\frac{(m-1)}{\rho_t}}\right)$$

$$q_f^{n+\frac{m}{\rho_t}}\Big|_{\gamma_h} = P(q_c^n, q_c^{n+1})$$

3. $q_c^{n+1}\Big|_{\omega_H} = R(q_f^{n+1})$

Outline

Time stepping issues: solution of a linear system

Let's suppose we have to solve:

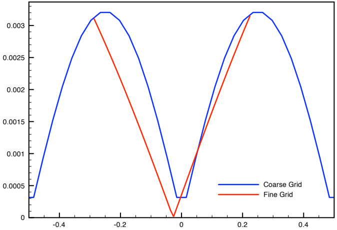
$$\mathcal{A}v = \mathcal{B}, \qquad \Delta q = \sin(x)$$

a) $\mathcal{A}_c v_c = \mathcal{B}_c$ on Ω_H

$$b) \begin{cases} \mathcal{A}_f v_f = \mathcal{B}_f \text{ on } \omega_h \\ v_{f_{|\gamma_h}} = P v_c \end{cases}$$

Naive approach

One way



Coarse and fine grid errors:
Naive approach

Time stepping issues: solution of a linear system

Let's suppose we have to solve:

$$\mathcal{A}v = \mathcal{B}, \qquad \Delta q = \sin(x)$$

$$a)\ \mathcal{A}_c v_c = \begin{cases} R\mathcal{B}_f & \text{in } \omega_H \\ \mathcal{B}_c & \text{in } \Omega_H \backslash \omega_H \end{cases}, \qquad b)\ \begin{cases} \mathcal{A}_f v_f = \mathcal{B}_f \text{ on } \omega_h \\ v_{f_{|\gamma_h}} = P v_c \end{cases}$$

Update of the right hand side

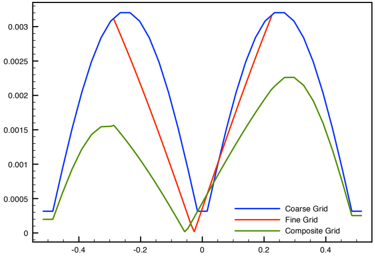
Time stepping issues: solution of a linear system

Let's suppose we have to solve:

$$\mathcal{A}v = \mathcal{B}, \qquad \Delta q = \sin(x)$$

$$\begin{pmatrix} \mathcal{A}_c & 0 \\ 0 & \mathcal{A}_f \\ \mathcal{A}_{c\gamma} & \mathcal{A}_{f\gamma} \end{pmatrix} \begin{pmatrix} v_c|_{\Omega_H \backslash \omega_H} \\ v_f \end{pmatrix} = \begin{cases} \mathcal{B}_c & \text{in } \Omega_H \backslash \omega_H \\ \mathcal{B}_f & \text{on } \omega_h \\ \mathcal{B}_\gamma & \text{in } \gamma_h \end{cases}$$

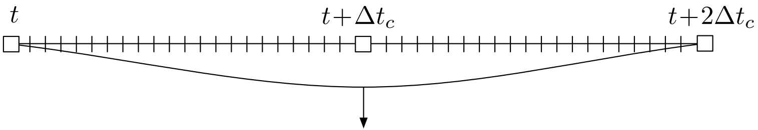
Multiresolution system on a composite grid



Coarse, fine and composite grid errors

Time stepping issues: split/explicit free surface

Barotropic time steps:



Filtering :

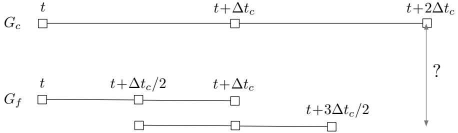
$$U(t+\Delta t_c) = \int_t^{t+2\Delta t_c} g(\xi - t)u(\xi)\mathrm{d}\xi$$

$$\int_t^{t+2\Delta t_c} g(\xi - t)\mathrm{d}\xi = 1, \quad \int_t^{t+2\Delta t_c} \xi g(\xi - t)\mathrm{d}\xi = t + \Delta t_c$$

One Way approach: coupling at the baroclinic level

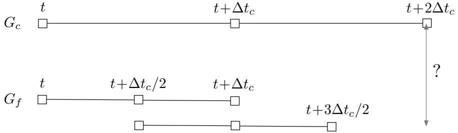
Time stepping issues: split/explicit free surface

How to perform the coupling at the barotropic level ?



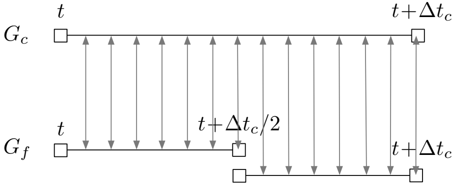
Time stepping issues: split/explicit free surface

How to perform the coupling at the barotropic level ?



Exchange between intermediate filtered quantities:

$$U(t+\alpha\Delta t_c) = \int_t^{t+2\alpha\Delta t_c} \frac{1}{\alpha} g\left(\frac{\xi - t}{\alpha}\right) u(\xi)\mathrm{d}\xi$$

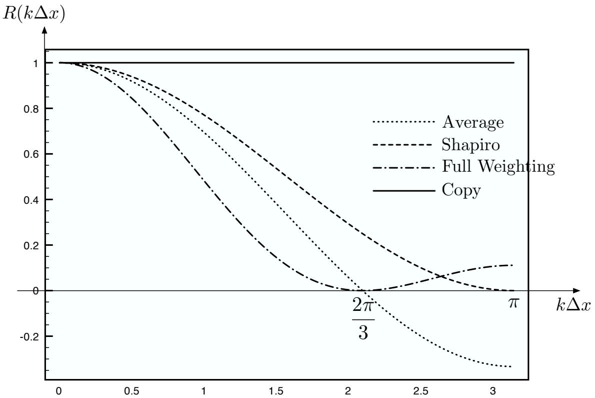


Outline

Update schemes

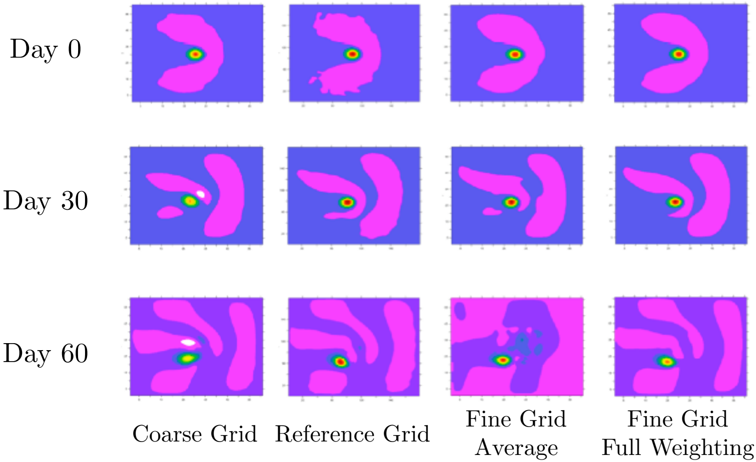
- Maximize the transfer of information for scales well resolved on the coarse grid
- Filter out the small scales

Update schemes



Copy	$q^c = q^f_j$
Average	$q^c = \frac{1}{3}\left(q^f_{j-1} + q^f_j + q^f_{j+1}\right)$
Shapiro	$q^c = \frac{1}{4}\left(q^f_{j-1} + 2q^f_j + q^f_{j+1}\right)$
Full Weighting	$q^c = \frac{1}{9}\left(q^f_{j-2} + 2q^f_{j-1} + 3q^f_j + 2q^f_{j+1} + q^f_{j+2}\right)$

Update schemes: Baroclinic Vortex



Outline

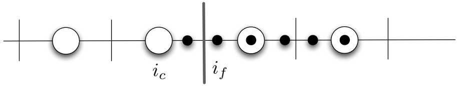
The conservation problem

- Maintain conservation
- Quantify artificial loss

The conservation problem

Let us consider a one dimensional domain and q, a solution of the following equation written in conservative form

$$\frac{\partial q}{\partial t} + \frac{\partial g(q)}{\partial x} = 0, \qquad g(q) = u_0\, q$$



$$q_{i_c}^{c,n+1} = q_{i_c}^{c,n} - \frac{\Delta t_c}{\Delta x_c}\left(g_{i_c}^{n} - g_{i_c-1}^{n}\right)$$

Coarse grid

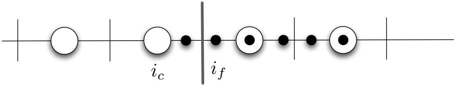
$$q_{i_f}^{f,n+1/2} = q_{i_f}^{f,n} - \frac{\Delta t_f}{\Delta x_f}\left(g_{i_f}^{n} - g_{i_f-1}^{n}\right)$$
$$q_{i_f}^{f,n+1} = q_{i_f}^{f,n+1/2} - \frac{\Delta t_f}{\Delta x_f}\left(g_{i_f}^{n+1/2} - g_{i_f-1}^{n+1/2}\right)$$

Fine grid

The conservation problem

Let us consider a one dimensional domain and q, a solution of the following equation written in conservative form

$$\frac{\partial q}{\partial t} + \frac{\partial g(q)}{\partial x} = 0, \qquad g(q) = u_0\, q$$



Composite grid approach:

$$Q^n = \sum_{-\infty}^{i_c} \Delta x_c q_i^{c,n} + \sum_{i_f}^{+\infty} \Delta x_f q_i^{f,n} \quad \left(\neq \sum_{-\infty}^{+\infty} \Delta x_c q_i^{c,n} \right)$$

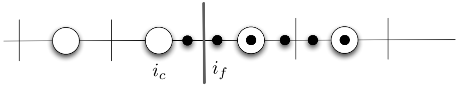
$$Q^{n+1} = Q^n - \left[\Delta t_c\, g_{i_c}^n - \Delta t_f \left(g_{i_f-1}^n + g_{i_f-1}^{n+1/2} \right) \right] \neq Q^n$$

Artificial loss of conservation

The conservation problem

Let us consider a one dimensional domain and q, a solution of the following equation written in conservative form

$$\frac{\partial q}{\partial t} + \frac{\partial g(q)}{\partial x} = 0, \qquad g(q) = u_0\, q$$



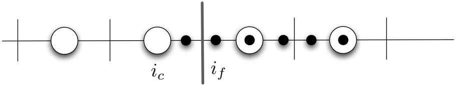
Flux correction:

$$q_{i_c}^{c,n+1,\star} = q_{i_c}^{c,n+1} + \frac{1}{\Delta x_c}\left[\Delta t_c\, g_{i_c}^{n} - \Delta t_f \left(g_{i_f-1}^{n} + g_{i_f-1}^{n+1/2}\right)\right]$$

The conservation problem

Let us consider a one dimensional domain and q, a solution of the following equation written in conservative form

$$\frac{\partial q}{\partial t} + \frac{\partial g(q)}{\partial x} = 0, \qquad g(q) = u_0\, q$$



Stability issues: g computed with centered schemes

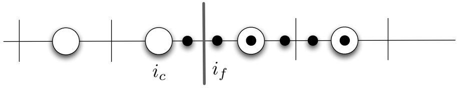
$$q_{i_c}^{c,n+1,\star} = q_{i_c}^{c,n+1} + \underbrace{\frac{1}{\Delta x_c}\left[\Delta t_c\, g_{i_c}^{n} - \Delta t_f \left(g_{i_f-1}^{n} + g_{i_f-1}^{n+1/2}\right)\right]}_{\Delta t_c \left(\frac{1}{9}\Delta x_c\, u_0\, \frac{\partial^2 q}{\partial x^2}\right)}$$

First order accurate

The conservation problem

Let us consider a one dimensional domain and q, a solution of the following equation written in conservative form

$$\frac{\partial q}{\partial t} + \frac{\partial g(q)}{\partial x} = 0, \qquad g(q) = u_0\, q$$



Stability issues: g computed with 3rd order upwind schemes

$$q_{i_c}^{c,n+1,\star} = q_{i_c}^{c,n+1} + \underbrace{\frac{1}{\Delta x_c}\left[\Delta t_c\, g_{i_c}^{n} - \Delta t_f \left(g_{i_f-1}^{n} + g_{i_f-1}^{n+1/2}\right)\right]}_{\Delta t_c \left(-\frac{13}{216}(\Delta x_c)^2 u_0\, \frac{\partial^3 q}{\partial x^3}\right)}$$

Outline

Sponge layer

- Maintain a strong consistency between high and coarse resolution solutions in the area where solutions interact (i.e. near the common interface)
- Prevent waves reflection

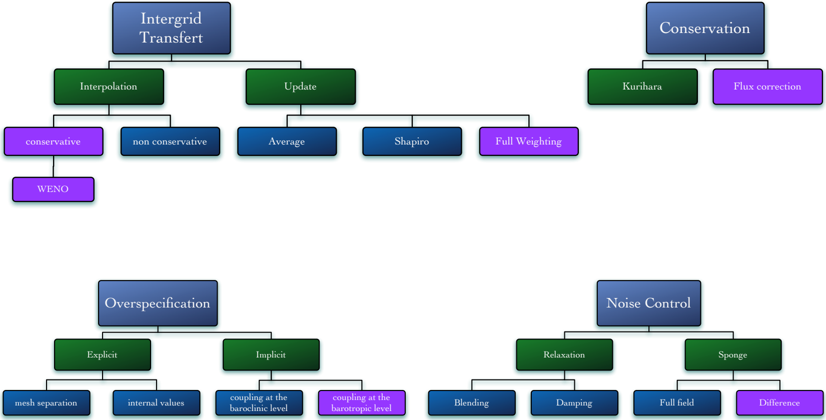
Sponge layer

- Maintain a strong consistency between high and coarse resolution solutions in the area where solutions interact (i.e. near the common interface)
- Prevent waves reflection

$$
\begin{aligned}
\frac{\partial q_f}{\partial t} &= \ldots + (-1)^{n+1}(\Delta)^n \left[\mu_{x,\partial\omega}(q_f - Pq_c)\right] \\
&= \ldots + (-1)^{n+1}(\Delta)^n \left[\mu_{x,\partial\omega} \underbrace{(I - PR)}_{\text{filter}} q_f)\right]
\end{aligned} \tag{1}
$$

Outline

Roms_Agrif: summary



→ integration in Roms_Agrif 2.0

Applications

Test cases similar to Penven et al, 2006, Ocean Modelling

- Baroclinic vortex
- USWC15km-5km

Applications

Baroclinic Vortex

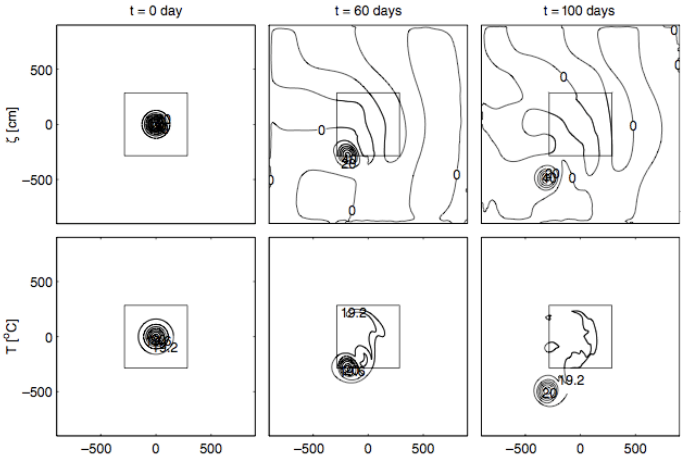
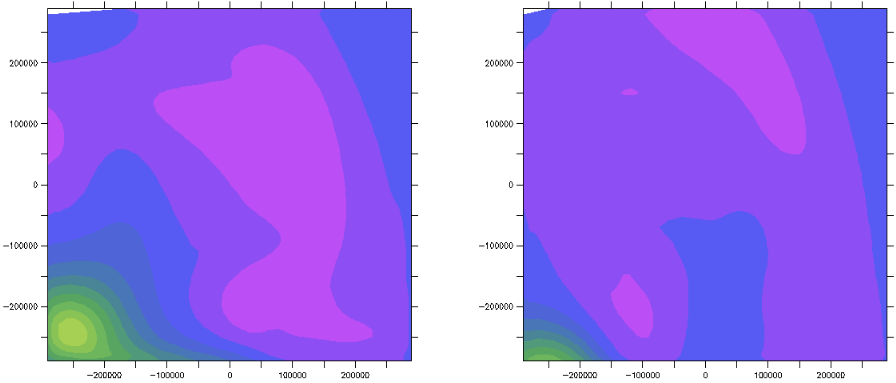


Fig. 1. Reference solution (V1) for the baroclinic vortex for days 0, 60 and 100. Top: sea surface elevation [cm], the contour interval is 10 cm. Bottom: sea surface temperature [°C], the contour interval is 0.2 °C. The box represents the embedded domain, in this case using the same resolution as the parent grid (10 km).

Applications

Baroclinic Vortex



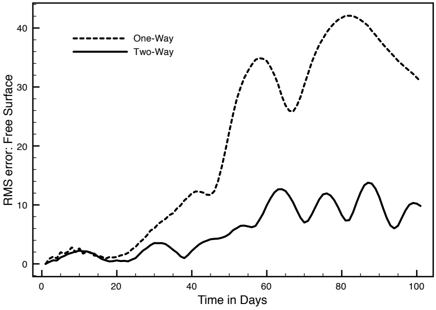
Free surface on the high resolution domain after 70 days: One-way (left), Two-way (right)

Applications

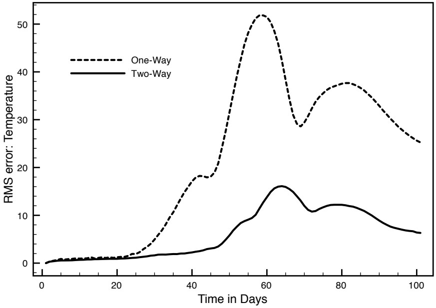
Baroclinic Vortex

Applications

Baroclinic Vortex



RMS error: Free surface



RMS error: Temperature

Applications

USWC15-5