Estimating Semivariograms to Build Correlation Matrices for *J*

Ralph F. Milliff and Jan Morzel
NWRA/CoRA Boulder, CO

Brian Powell and Dax Matthews
Univ. Hawaii, Honolulu, HI

$$J = \mathbf{f} \bullet C_f^{-1} \bullet \mathbf{f} + \mathbf{i} \circ C_i^{-1} \circ \mathbf{i} + \mathbf{b} * C_b^{-1} * \mathbf{b}$$

where C_f is $N_{2D} \times N_{2D}$ for each forcing field, C_i is $N_{3D} \times N_{3D}$ for each state vector, and C_b is $N_{\partial\Omega} \times N_{\partial\Omega}$ for each state variable specified on the domain boundary $\partial\Omega$.

- Semivariograms (empirical, theoretical)
- Generalized Diffusion Equation methods

Banerjee et al., 2004: Hierarchical modeling and analysis for spatial data. *Monographs on Statistics and Applied Probability 101*, Chapman and Hall/CRC Press, Boca Raton, FL

Matthews, D. and B. Powell; 2010 in preparation.

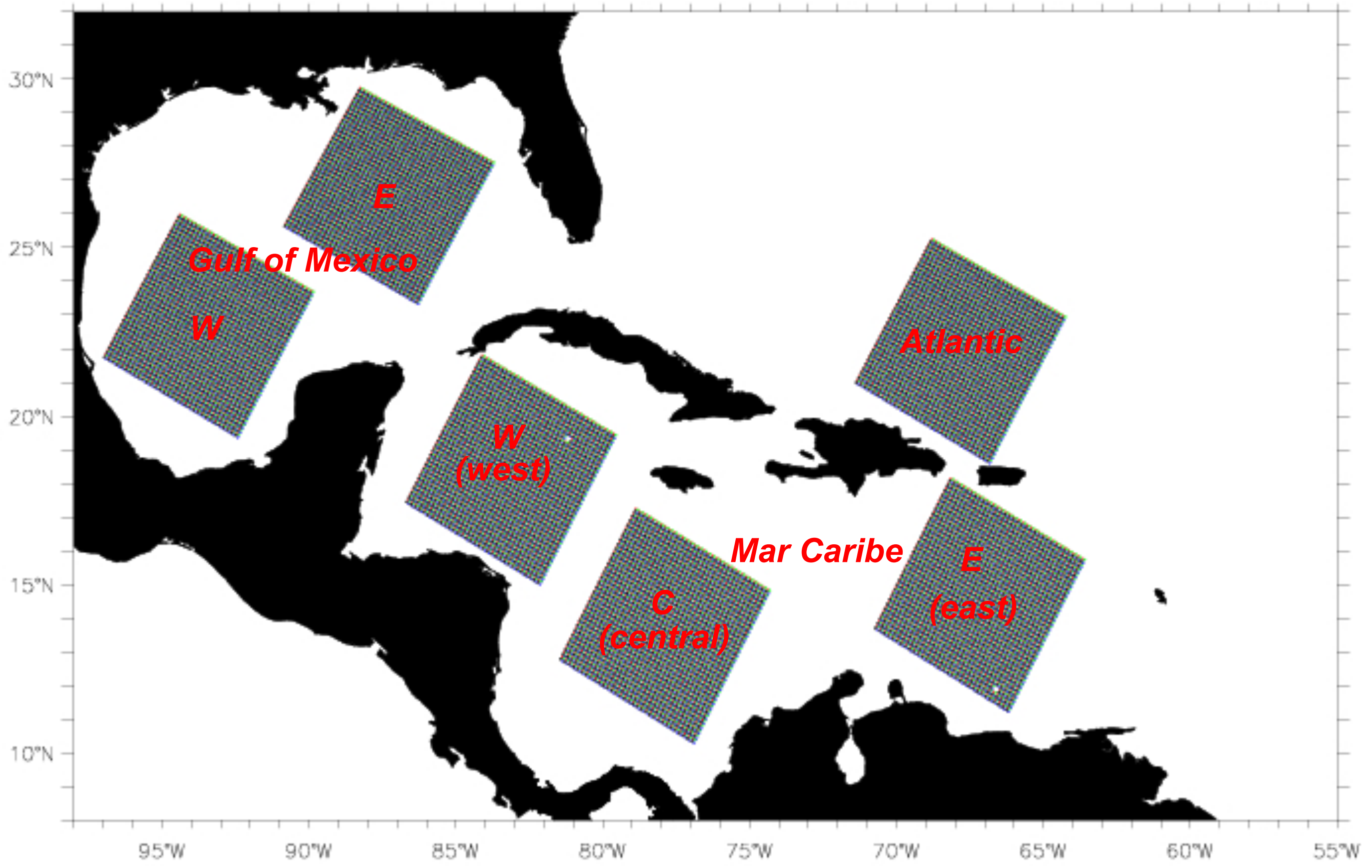
Powell et al., 2008: 4DVAR data assimilation in the Intra-Americas Sea with the Regional Ocean Modeling System. *Ocean Modelling*, **23,** 130-145.

Powell et al., 2009: Near real-time assimilation and prediction in the Intra-Americas Seas with the Regional Ocean Modeling System. *Dyn. Atmos. Oceans*, **48**, 46-68.

Banerjee et al., 2004: Hierarchical modeling and analysis for spatial data. *Monographs on Statistics and Applied Probability 101*, Chapman and Hall/CRC Press, Boca Raton, FL

Weaver, A. and P. Courtier, 2001: Correlation modeling on the sphere using generalized diffusion equation. *Q.J.R. Meteorol. Soc.*, **127**, 1815-1846.

Regional Domains for Semivariogram Calculations



32x32 ROMS grid locations for each box

Sep 2003 (hurricane season), Apr 2004; QuikSCAT, NCEP, blended QSCAT-NCEP

Semivariogram Estimate for spatial data

$$\hat{\gamma}(h) = \frac{1}{N}\sum_{i=1}^{N}\left[\eta(x_i + h) - \eta(x_i)\right]^2$$

Relation to Spatial Covariance

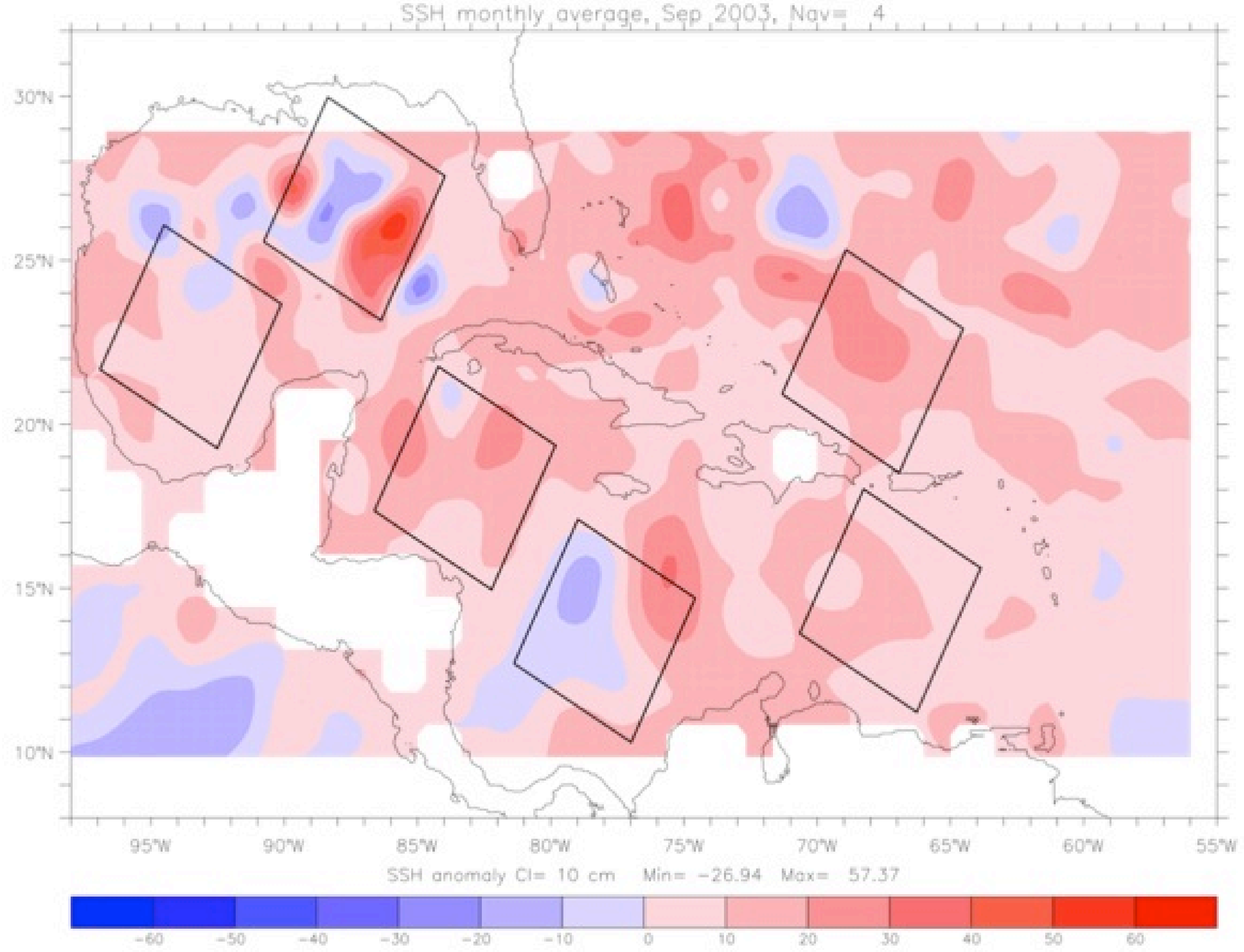
$$\begin{aligned}
2\gamma(h) &= \mathrm{Var}(\eta(x+h) - \eta(x)) \\
&= \mathrm{Var}(\eta(x+h)) + \mathrm{Var}(\eta(x)) - 2\mathrm{Cov}(\eta(x+h), \eta(x)) \\
&= C(0) + C(0) - 2C(h) \\
&= 2\left[C(0) - C(h)\right] \\
\\
\gamma(h) &= C(0) - C(h)
\end{aligned}$$

Correlation Model Fit (exponential model)

$$C(h) = \sigma^2[\,1 - \exp(-\frac{h}{\theta})\,]$$

where σ^2 is the *sill* and θ is the *range*

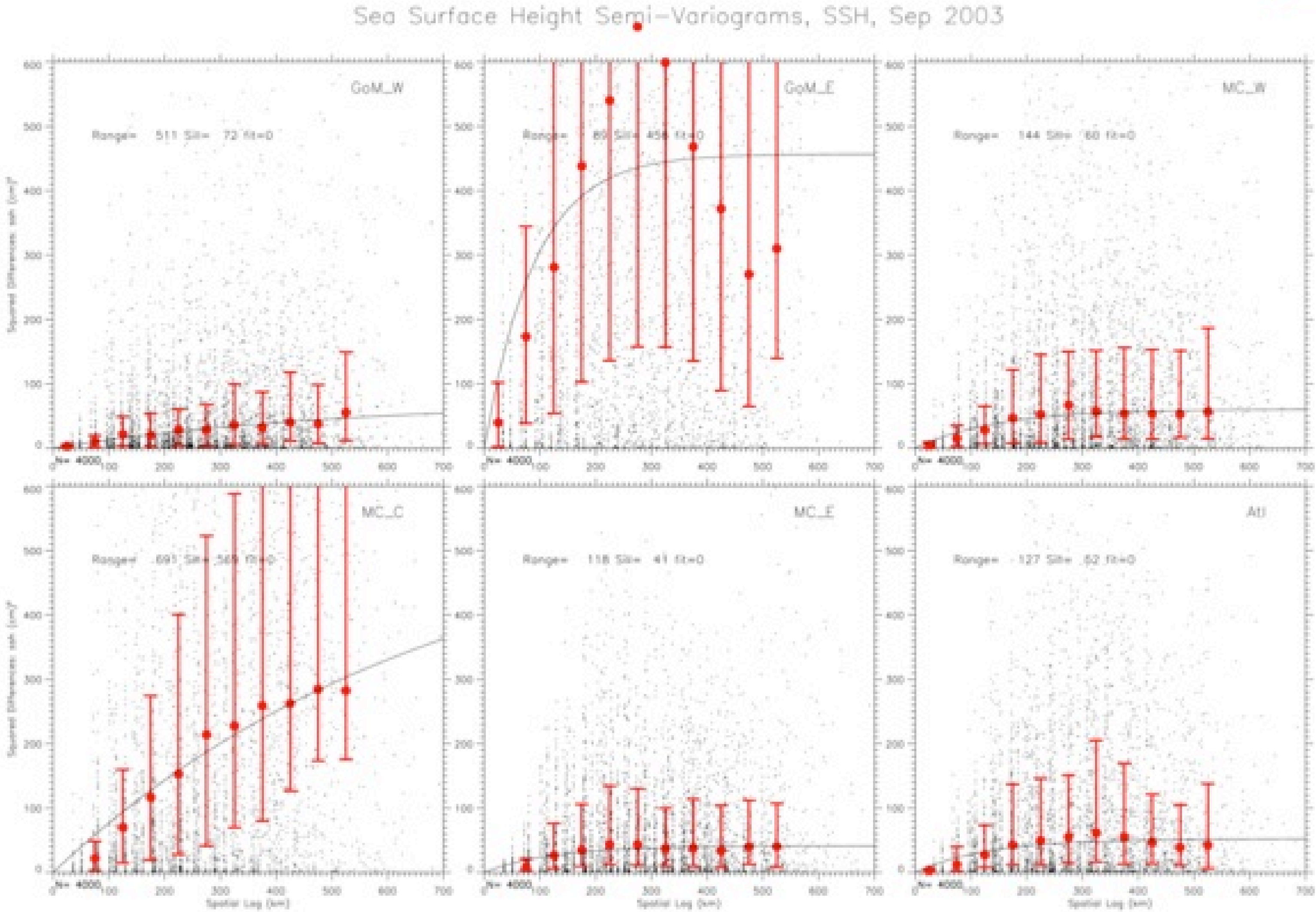
Monthly Average SSH Anomaly from AVISO *(September 2003)*



σ^2 *"sill"*

θ *"range"*

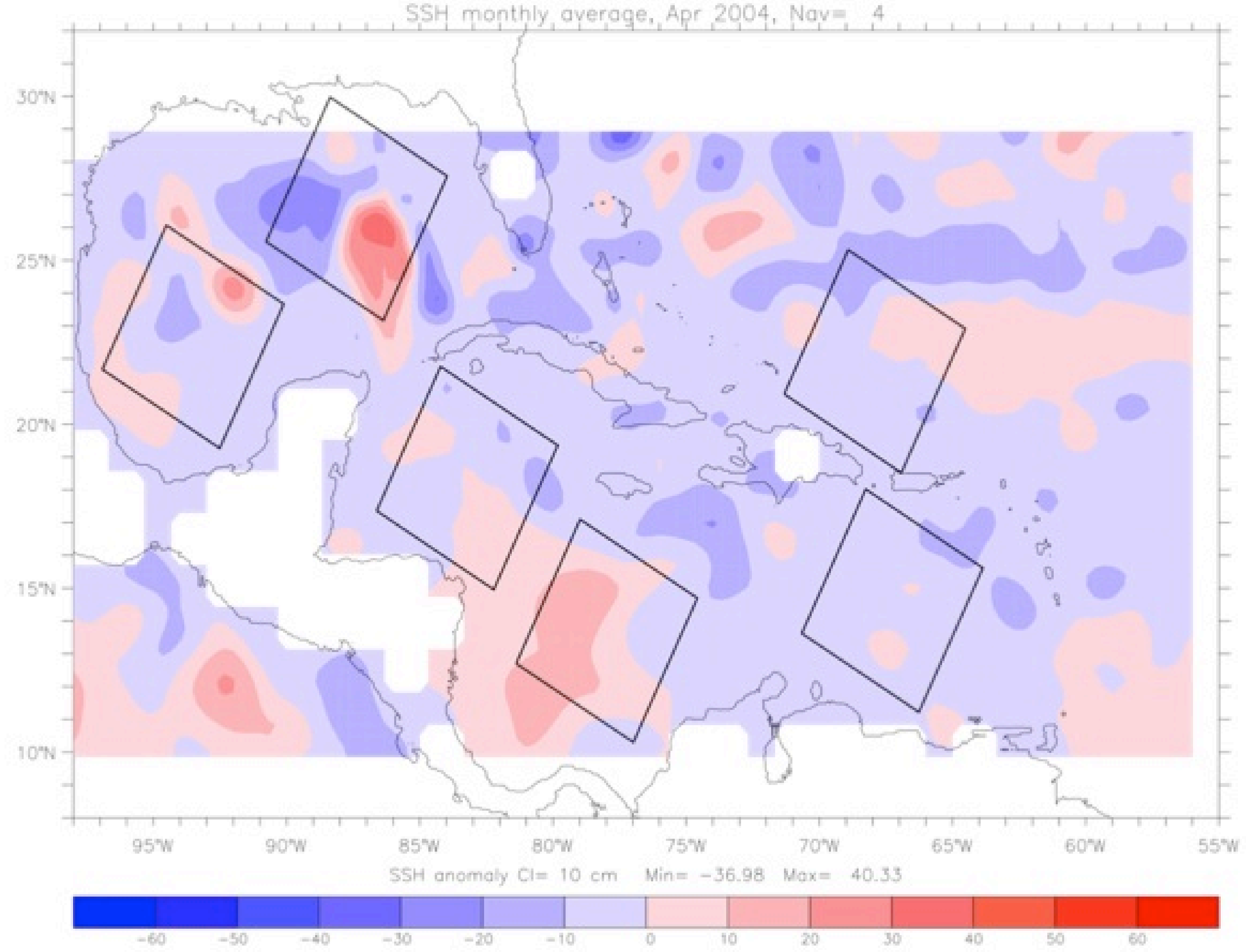
SSH; September 2003



$\hat{\gamma}(h)$

$$h = [x_i - (x_i + h)]$$

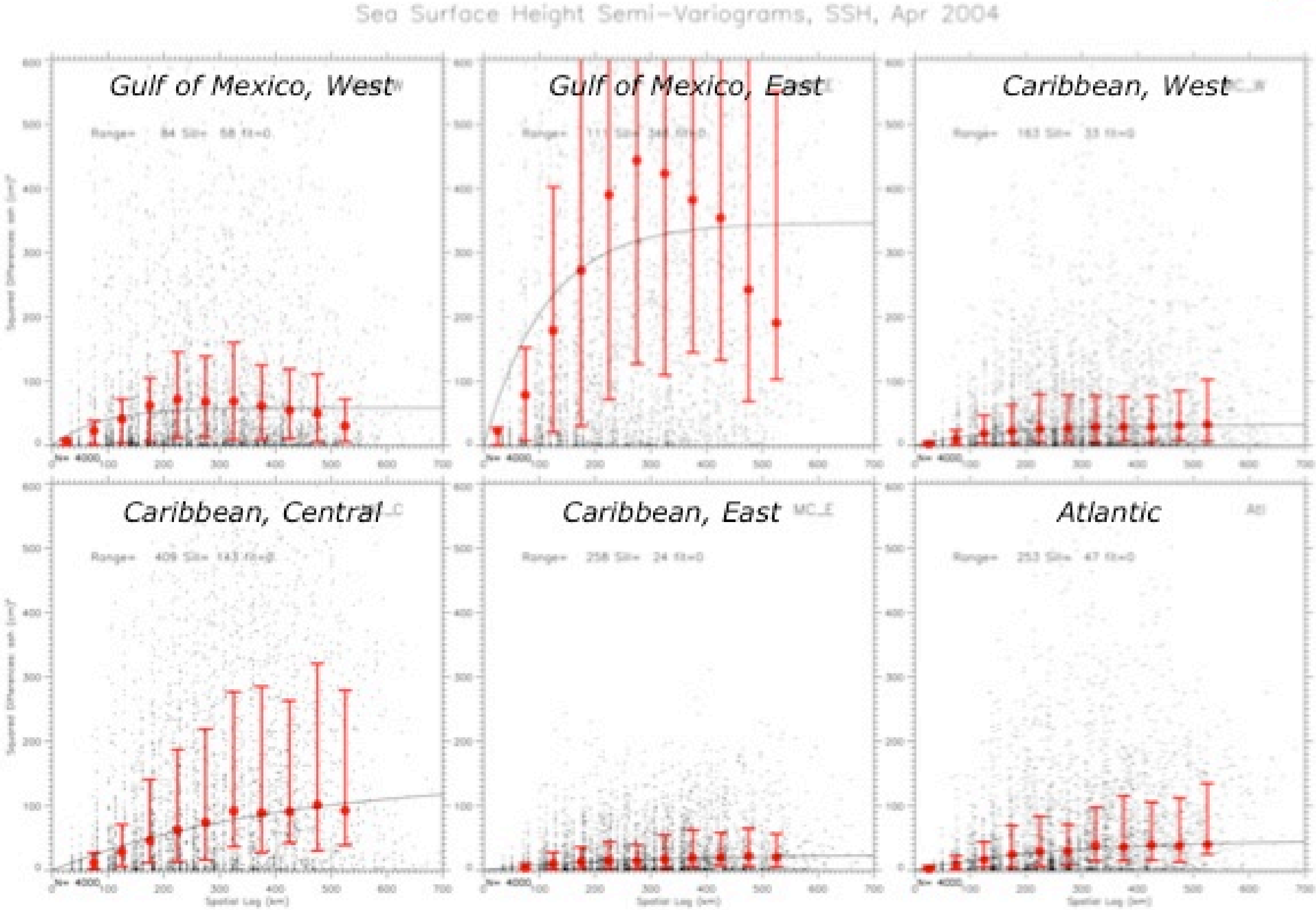
Monthly Average SSH Anomaly from AVISO *(April 2004)*



σ^2 *"sill"*

θ *"range"*

SSH; April 2004

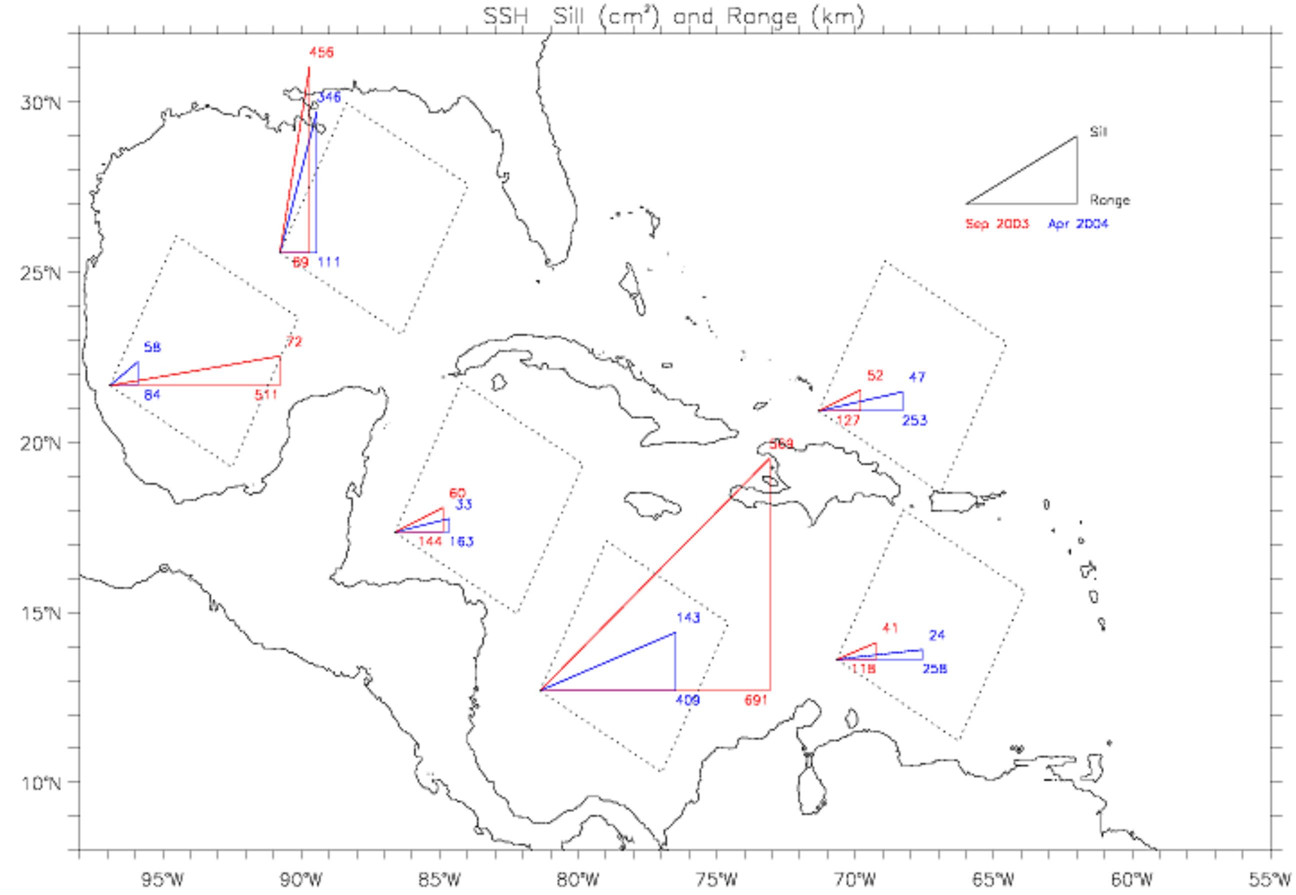


$$\hat{\gamma}(h)$$

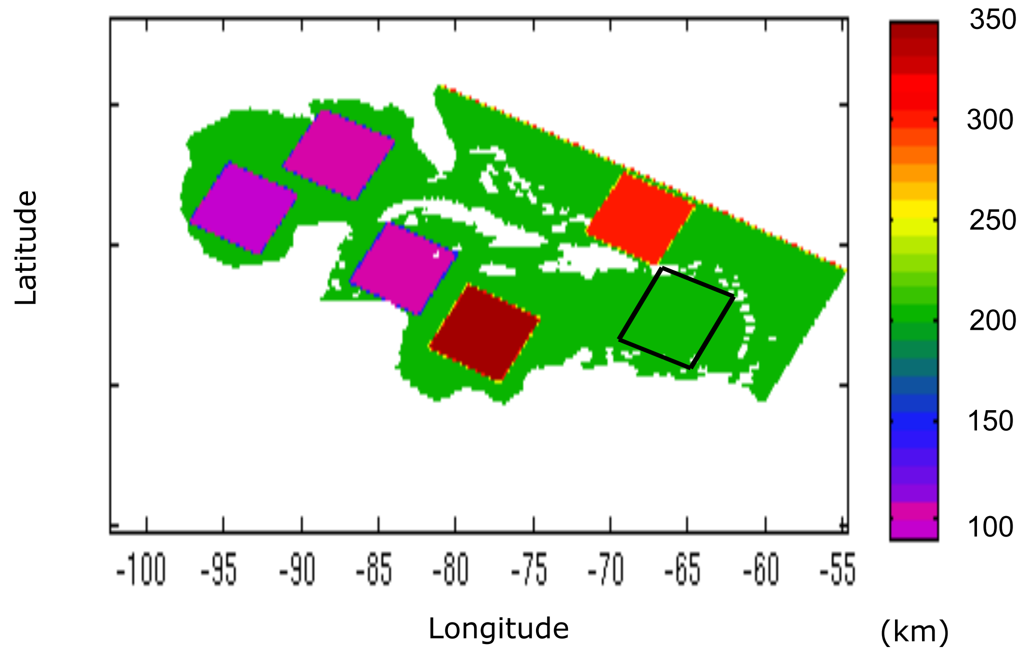
$$h = [x_i - (x_i + h)]$$

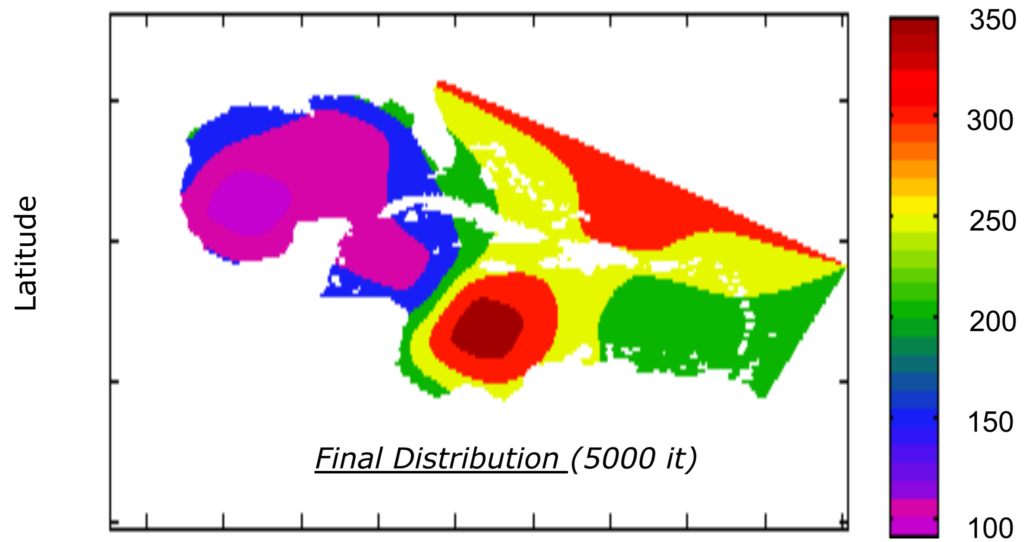
Range and Sill Parameters from Semivariogram Analysis

SSH Anomalies; Sep 2003 and Apr 2004



Distribute Covariance Parameters: *Laplacian Smoother (Range Example)*





Final Distribution (5000 it)

$$\theta^t = \nabla^2\theta^{t-1} + r\theta^{t_0}$$

$$r = \begin{cases} 1 & \text{if } \theta_j^t \le \theta_j^{t_0} - \epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\epsilon = 0.5\,km$$

θ^{t_0} is the initial distribution, $\theta_j,\ j = 1, 2, \ldots, 6$ are the center values for each sub-domain.

Generalized Diffusion Equation Methods (Weaver and Courtier, 2001)

for a field variable η, model the correlation at each point in the field as:

$$\eta(t_M) = \mathbf{L}_P \, \eta(t_0)$$

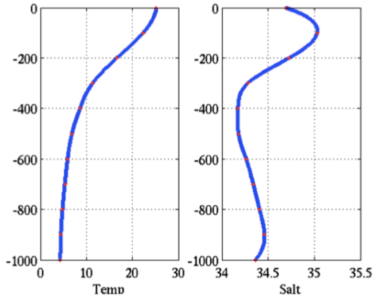
where

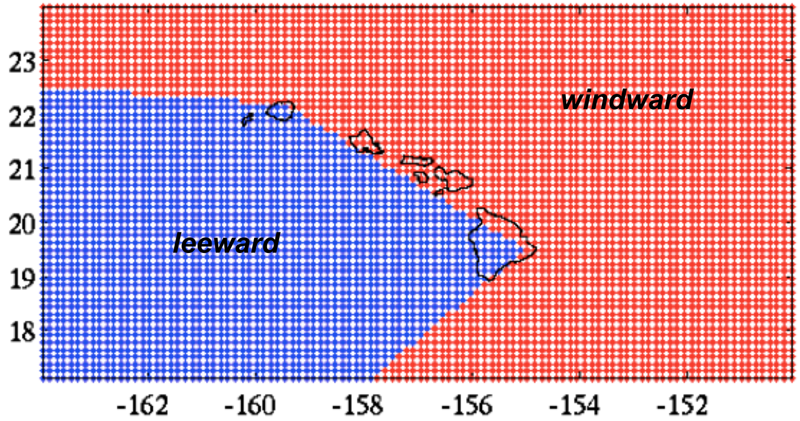
$$\mathbf{L}_P = \left[\mathbf{I} - \sum_{p=1}^{P} \kappa_p \Delta t (-\mathbf{D})^P \right]^M$$

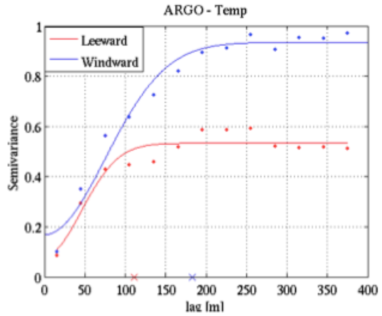
$$\eta(t_m) = \eta(t_{m-1}) - \sum_{p=1}^{P} \kappa_p \Delta t (-\nabla)^P \eta(t_{m-1})$$

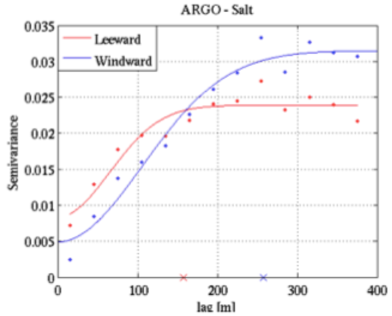
Hawaiian Island Domain Example: *SST, SSH, T(z), S(z)*

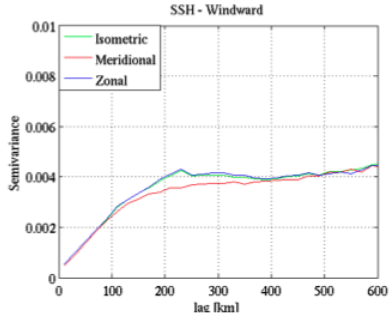
Matthews and Powell (2010)

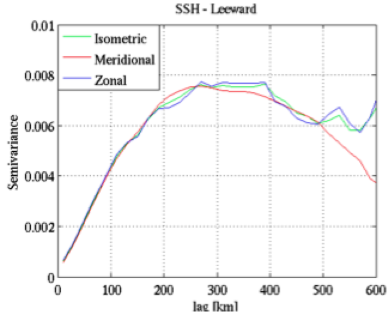


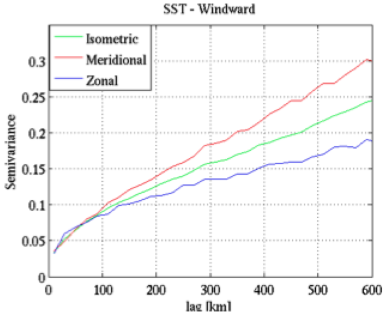


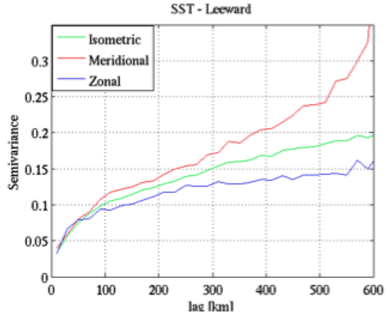

ARGO - Temp

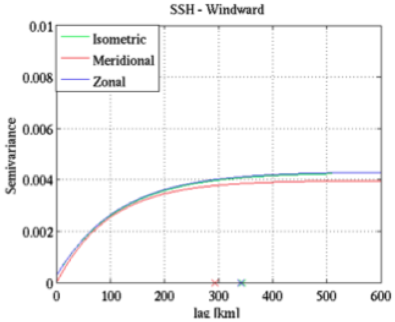

ARGO - Salt


SSH - Windward


SSH - Leeward


SST - Windward


SST - Leeward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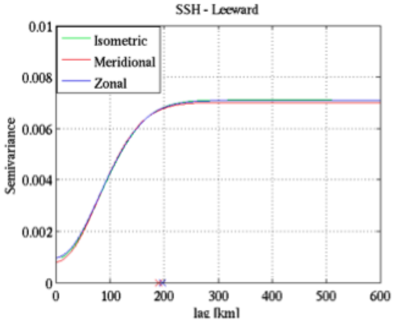
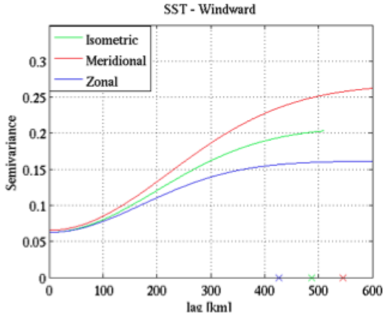
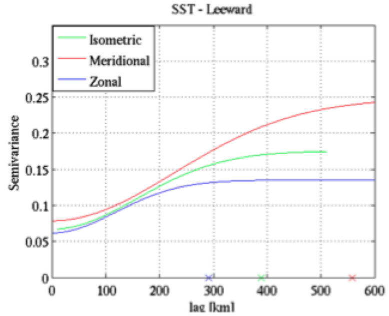
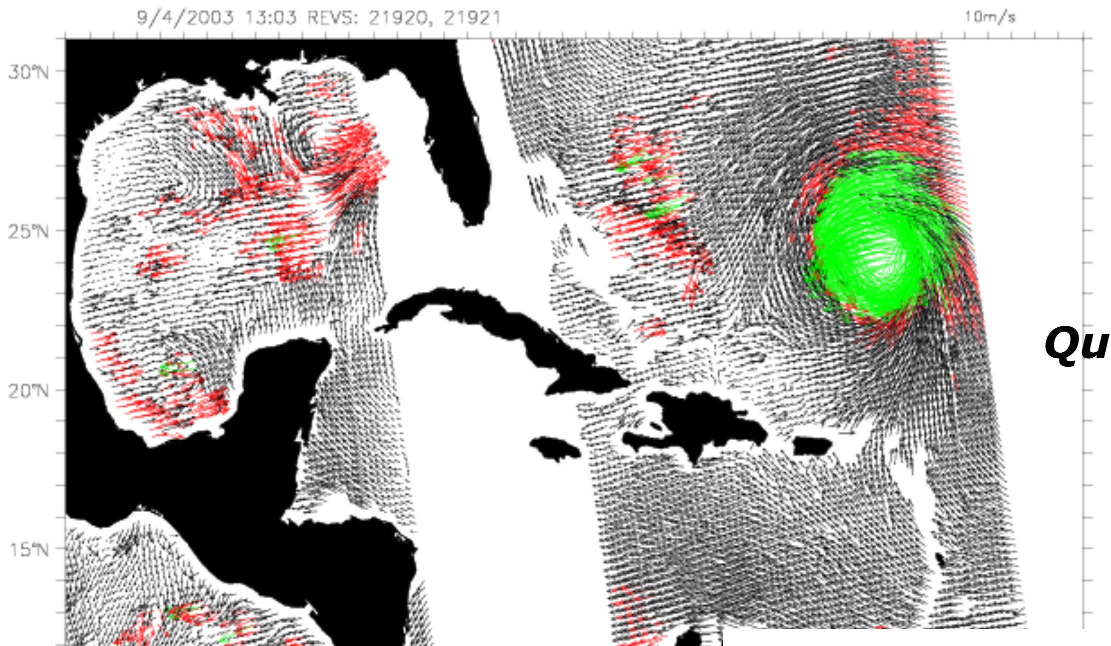

SSH - Leeward

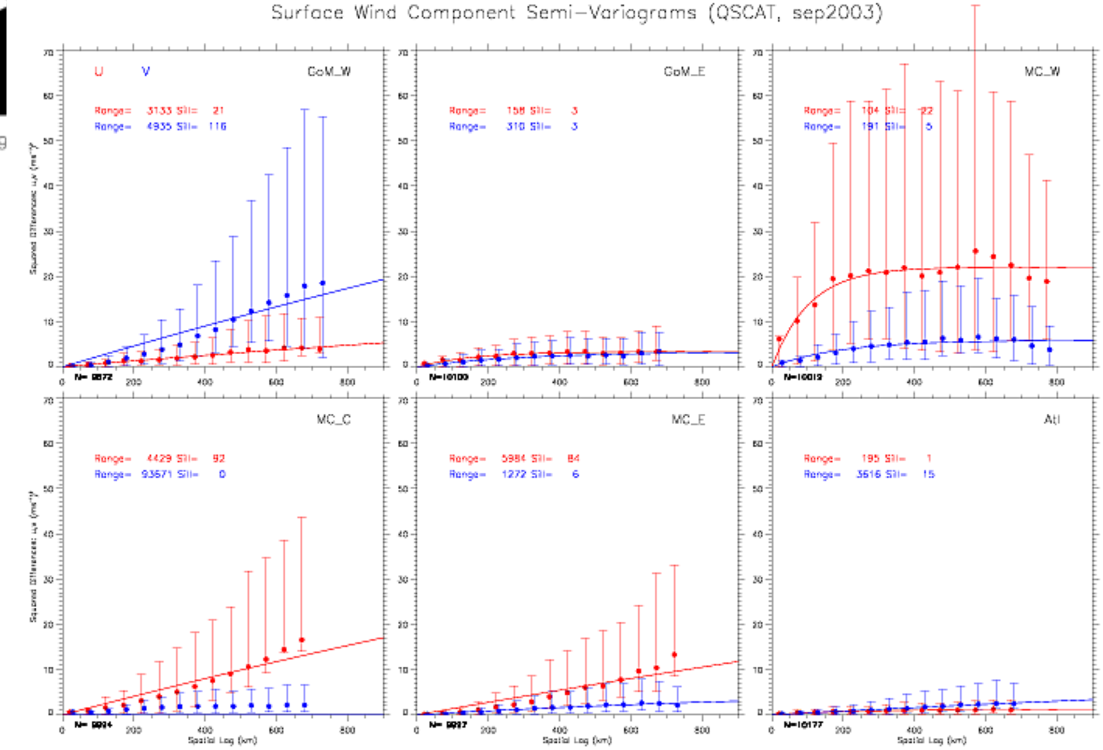

SST - Windward


SST - Leeward

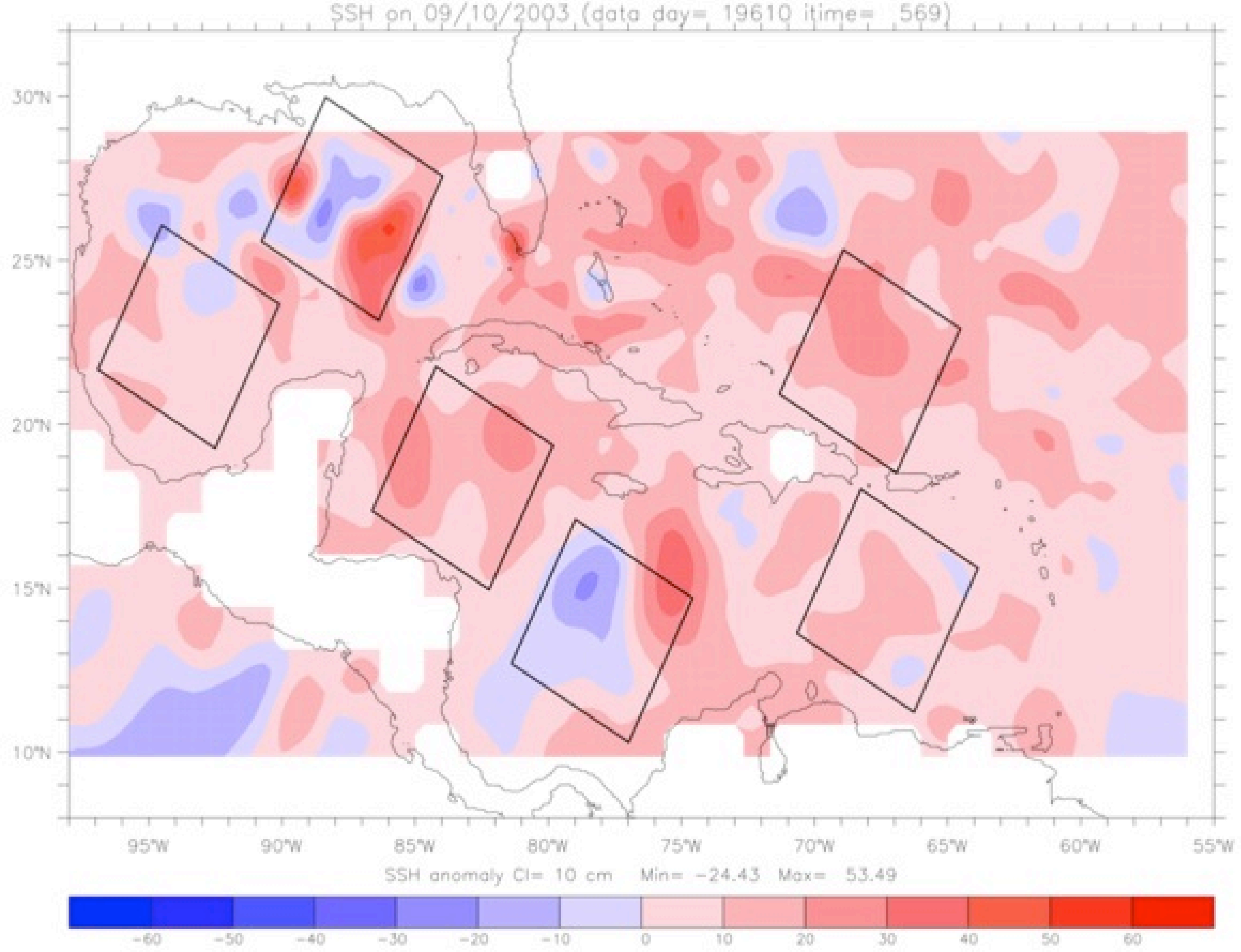


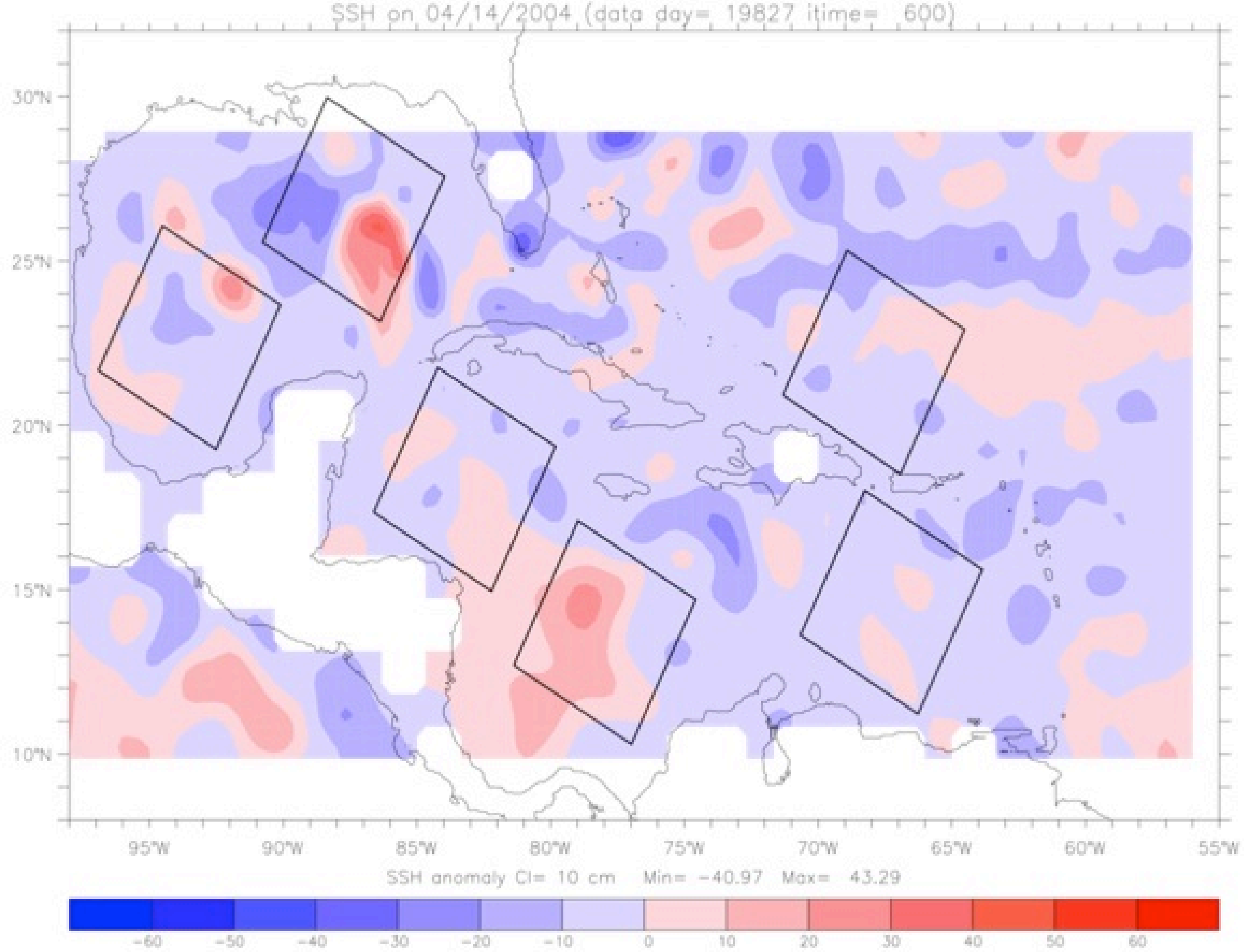
Semivariogram Analysis:
QuikSCAT Surface Vector Wind

September 2003
"Hurricane Season"



EXTRA SLIDES


SSH on 09/10/2003 (data day= 19610 itime= 569)
30°N
25°N
20°N
15°N
10°N
95°W
90°W
85°W
80°W
75°W
70°W
65°W
60°W
55°W
SSH anomaly CI= 10 cm Min= -24.43 Max= 53.49
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60


SSH on 04/14/2004 (data day= 19827 itime= 600)
30°N
25°N
20°N
15°N
10°N
95°W
90°W
85°W
80°W
75°W
70°W
65°W
60°W
55°W
SSH anomaly CI= 10 cm Min= -40.97 Max= 43.29
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60