Tutorial 4:
Standard Deviations for *Prior* Error Correlations

Prior Error Covariance Modeling

Recall: $\mathbf{B_x} = \mathbf{K_b \Sigma C \Sigma^T K_b^T}$

where Σ is a diagonal matrix of *prior* error standard deviations.

Several possible ways to estimate Σ:

(i) Climatological variance:

Consider an ensemble of randomly chosen ocean states with covariance B_c.

Before doing any data assimilation, the *prior* x_b will also be a valid member of this ensemble, so we can use B_c as an estimate of the 4D-Var *prior* error covariance B_x.

Prior Error Covariance Modeling

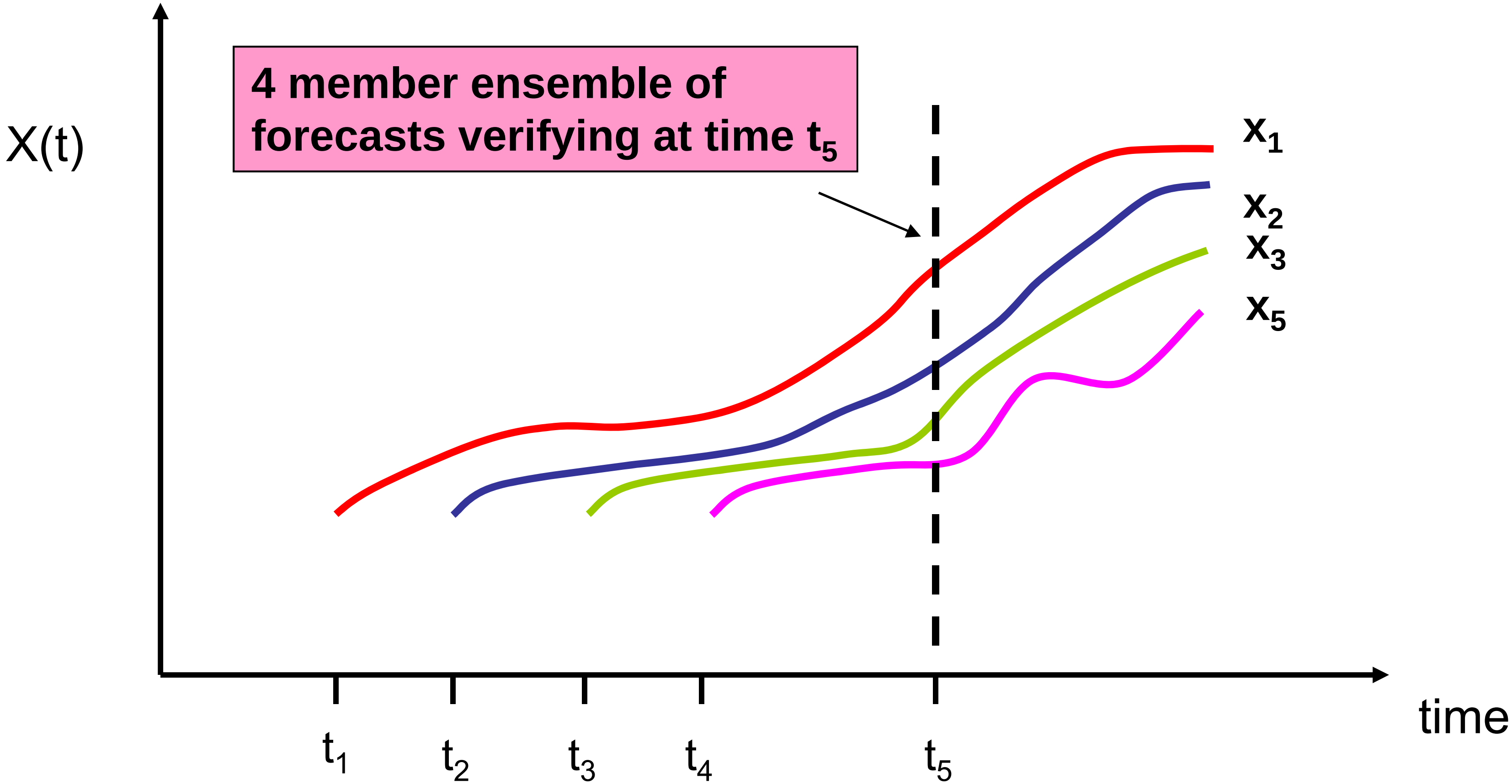
(ii) Variance from overlapping forecast ensembles (the "NMC method"):

After a few cycles of 4D-Var, the ROMS ocean state estimate will be more accurate than climatology, so B_c will not be an appropriate estimate for the *prior* error covariance B_x of x_b.

In this case, an ensemble of overlapping forecasts that all verify at the initial time of the 4D-Var analysis can be used to estimate B_x, if one is available.

The NMC Method

(Parish and Derber, 1992)



$$\mathbf{B}_{\mathbf{x}} \approx \frac{1}{4}\sum_{i=1}^{4}\left(\mathbf{x}_i(t_5)-\overline{\mathbf{x}}(t_5)\right)\left(\mathbf{x}_i(t_5)-\overline{\mathbf{x}}(t_5)\right)^{\mathrm{T}}$$

Used in ROMS 4D-Var by Powell et al (2008)

4D-Var Standard Deviation Files

- **Four different standard deviation NetCDF files are required in ROMS 4D-Var algorithms to convert modeled, prior error correlations (C) to error covariances (B_x):**
  - **Model error standard deviation file, if weak constraint**
  - **Initial conditions standard deviation file**
  - **Open boundary conditions standard deviation file, if ADJUST_BOUNDARY**
  - **Surface forcing standard deviation file, if ADJUST_WSTRESS and/or ADJUST_STFLUX**
- **These standard deviation NetCDF files are specified in 4D-Var input script as:**

```
STDnameM == ../Data/wc13_std_m.nc

STDnameI == ../Data/wc13_std_i.nc

STDnameB == ../Data/wc13_std_b.nc

STDnameF == ../Data/wc13_std_f.nc
```

Model Error and Initial Conditions Metadata

```
Variables:
. . .

        double zeta(ocean_time, eta_rho, xi_rho) ;
                zeta:long_name = "free-surface standard deviation" ;
                zeta:units = "meter" ;
                zeta:time = "ocean_time" ;
                zeta:coordinates = "lon_rho lat_rho ocean_time" ;
        double ubar(ocean_time, eta_u, xi_u) ;
                ubar:long_name = "vertically integrated u-momentum component standard deviation" ;
                ubar:units = "meter second-1" ;
                ubar:time = "ocean_time" ;
                ubar:coordinates = "lon_u lat_u ocean_time" ;
        double vbar(ocean_time, eta_v, xi_v) ;
                vbar:long_name = "vertically integrated v-momentum component standard deviation" ;
                vbar:units = "meter second-1" ;
                vbar:time = "ocean_time" ;
                vbar:coordinates = "lon_v lat_v ocean_time" ;
        double u(ocean_time, s_rho, eta_u, xi_u) ;
                u:long_name = "u-momentum component standard deviation" ;
                u:units = "meter second-1" ;
                u:time = "ocean_time" ;
                u:coordinates = "lon_u lat_u s_rho ocean_time" ;
        double v(ocean_time, s_rho, eta_v, xi_v) ;
                v:long_name = "v-momentum component standard deviation" ;
                v:units = "meter second-1" ;
                v:time = "ocean_time" ;
                v:coordinates = "lon_v lat_v s_rho ocean_time" ;
        double temp(ocean_time, s_rho, eta_rho, xi_rho) ;
                temp:long_name = "potential temperature standard deviation" ;
                temp:units = "Celsius" ;
                temp:time = "ocean_time" ;
                temp:coordinates = "lon_rho lat_rho s_rho ocean_time" ;
        double salt(ocean_time, s_rho, eta_rho, xi_rho) ;
                salt:long_name = "salinity standard deviation" ;
                salt:time = "ocean_time" ;
                salt:coordinates = "lon_rho lat_rho s_rho ocean_time" ;
```

Matlab script: matlab/4dvar/c_std.m

CDL script: Data/ROMS/CDL/s4dvar_std_i.cdl

Open Boundary Conditions Metadata

```
dimensions:

        xi_rho = 56 ;
        eta_rho = 55 ;
. . .
        IorJ = 56 ;
        boundary = 4 ;

variables:
. . .
        double zeta_obc(bry_time, boundary, IorJ) ;
                zeta_obc:long_name = "free-surface, open boundaries conditions standard deviation" ;
                zeta_obc:units = "meter" ;
                zeta_obc:time = "bry_time" ;
        double ubar_obc(bry_time, boundary, IorJ) ;
                ubar_obc:long_name = "vertically integrated u-momentum component, open boundaries conditions standard deviation" ;
                ubar_obc:units = "meter second-1" ;
                ubar_obc:time = "bry_time" ;
        double vbar_obc(bry_time, boundary, IorJ) ;
                vbar_obc:long_name = "vertically integrated v-momentum component, open boundaries conditions standard deviation" ;
                vbar_obc:units = "meter second-1" ;
                vbar_obc:time = "bry_time" ;
        double u_obc(bry_time, s_rho, boundary, IorJ) ;
                u_obc:long_name = "u-momentum component, open boundaries conditions standard deviation" ;
                u_obc:units = "meter second-1" ;
                u_obc:time = "bry_time" ;
        double v_obc(bry_time, s_rho, boundary, IorJ) ;
                v_obc:long_name = "v-momentum component, open boundaries conditions standard deviation" ;
                v_obc:units = "meter second-1" ;
                v_obc:time = "bry_time" ;
        double temp_obc(bry_time, s_rho, boundary, IorJ) ;
                temp_obc:long_name = "potential temperature, open boundaries conditions standard deviation" ;
                temp_obc:units = "Celsius" ;
                temp_obc:time = "bry_time" ;
        double salt_obc(bry_time, s_rho, boundary, IorJ) ;
                salt_obc:long_name = "salinity, open boundaries conditions standard deviation" ;
                salt_obc:time = "bry_time" ;

// global attributes:
. . .

                :boundary_index = "West=1, South=2, East=3, North=4"
```

Matlab script: matlab/4dvar/c_std_bry.m

CDL script: Data/ROMS/CDF/s4dvar_std_b.cdl

Surface Forcing Metadata

```
dimensions:

        xi_rho = 56 ;
        xi_u = 55 ;
        xi_v = 56 ;
        eta_rho = 55 ;
        eta_u = 55 ;
        eta_v = 54 ;
        s_rho = 30 ;
        ocean_time = UNLIMITED ; // (1 currently)

variables:
. . .
        double sustr(ocean_time, eta_u, xi_u) ;
                sustr:long_name = "surface u-momentum stress standard deviation" ;
                sustr:units = "newton meter-2" ;
                sustr:time = "ocean_time" ;
                sustr:coordinates = "lon_u lat_u ocean_time" ;
        double svstr(ocean_time, eta_v, xi_v) ;
                svstr:long_name = "surface v-momentum stress standard deviation" ;
                svstr:units = "newton meter-2" ;
                svstr:time = "ocean_time" ;
                svstr:coordinates = "lon_v lat_v ocean_time" ;
        double shflux(ocean_time, eta_rho, xi_rho) ;
                shflux:long_name = "surface net heat flux standard deviation" ;
                shflux:units = "watt meter-2" ;
                shflux:negative = "upward flux, cooling" ;
                shflux:positive = "downward flux, heating" ;
                shflux:time = "ocean_time" ;
                shflux:coordinates = "lon_rho lat_rho ocean_time" ;
        double ssflux(ocean_time, eta_rho, xi_rho) ;
                ssflux:long_name = "surface net salt flux (E-P)*SALT standard deviation" ;
                ssflux:units = "meter second-1" ;
                ssflux:time = "ocean_time" ;
                ssflux:coordinates = "lon_rho lat_rho ocean_time" ;


// global attributes:

                :type = "ROMS/TOMS 4D-Var surface forcing error covariance standard deviation" ;
                :title = "California Current System, 1/3 degree resolution (WC13)" ;
                :Conventions = "CF-1.4" ;
                :grd_file = "test/WC13/Data/wc13_grd.nc" ;
```

Matlab script: matlab/4dvar/c_std_frc.m

CDL script: Data/ROMS/CDF/s4dvar_std_f.cdl

Standard Deviation Matlab Scripts

There are several scripts in the matlab/4dvar repository to process standard deviations (https://www.myroms.org/svn/src):

- **c_std.m** **creates model error or initial conditions NetCDF file**
- **c_std_bry.m** **creates open boundary conditions NetCDF file**
- **c_std_frc.m** **creates surface forcing NetCDF file**

and associated drivers templates:

- **d_std.m**
- **d_std_unbalanced.m**
- **d_std_bry.m**
- **d_std_frc.m**

d_std.m

```
%  Set standard deviation NetCDF file. The file name is edited and the
%  month will be appended as *i_jan.nc:

STDfile = 'wc13_std_i.nc';

mstr = {'jan', 'feb', 'mar', 'apr', 'may', 'jun', ...
        'jul', 'aug', 'sep', 'oct', 'nov', 'dec'};

%  Set input grif file.

my_root = '/home/arango/ocean/toms/repository/test';

GRDfile = fullfile(my_root, 'WC13/Data', 'wc13_grd.nc');

%  Set input history files (string cell structure).

my_root = '/home/arango/ocean/toms/repository/test';

HISdir  = fullfile(my_root, 'WC13/STD/Data');

HISfile = dir(fullfile(HISdir, 'wc*.nc'));

nfiles = length(HISfile);

%----------------------------------------------------------------------------
%  Compute monthly averages and standard deviations.
%----------------------------------------------------------------------------

%  Extract first history file name.

HisFile1 = fullfile(HISdir, HISfile(1).name);
```

d_std_unbalanced.m

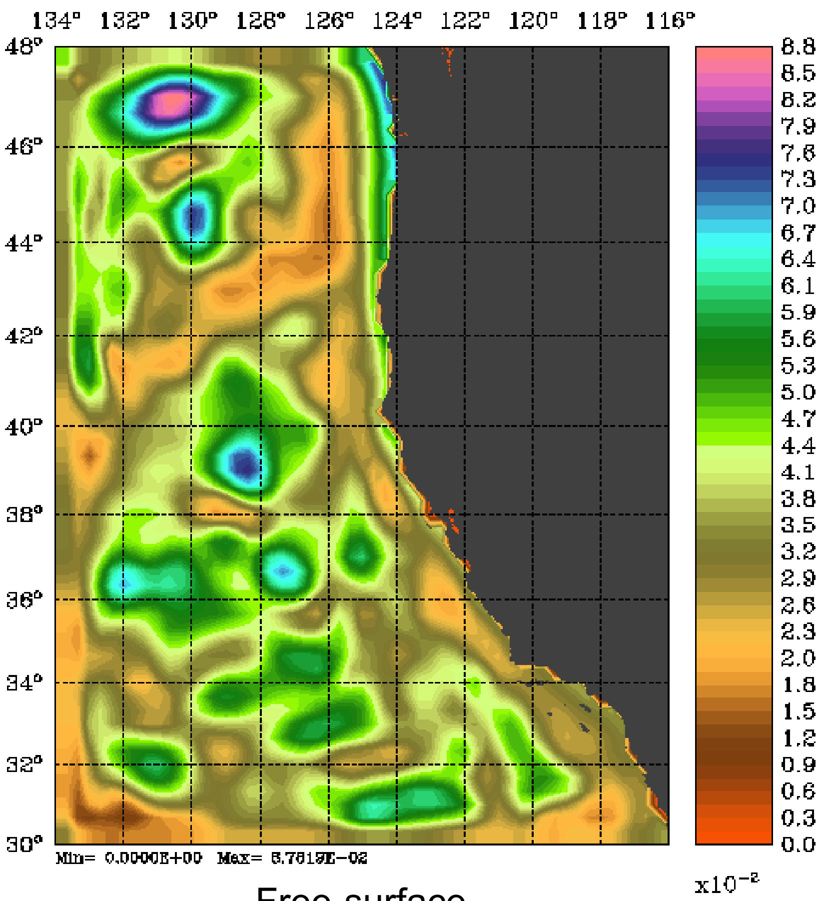
```
%  Set standard deviation NetCDF file. The file name is edited and the
%  month will be appended as *iu_jan.nc:

STDfile = 'wc13_std_iu.nc';

mstr = {'jan', 'feb', 'mar', 'apr', 'may', 'jun', ...
        'jul', 'aug', 'sep', 'oct', 'nov', 'dec'};

%  Set input grif file.

my_root = '/home/arango/ocean/toms/repository/test';

GRDfile = fullfile(my_root, 'WC13/Data', 'wc13_grd.nc');

%  Set input history files (string cell structure).

my_root = '/home/arango/ocean/toms/repository/test';

HISdir  = fullfile(my_root, 'WC13/STD/Data');

HISfile = dir(fullfile(HISdir, 'wc*.nc'));

nfiles = length(HISfile);

%----------------------------------------------------------------------------
%  Compute monthly averages and standard deviations.
%----------------------------------------------------------------------------

%  Extract first history file name.

HisFile1 = fullfile(HISdir, HISfile(1).name);
```

d_std_frc.m

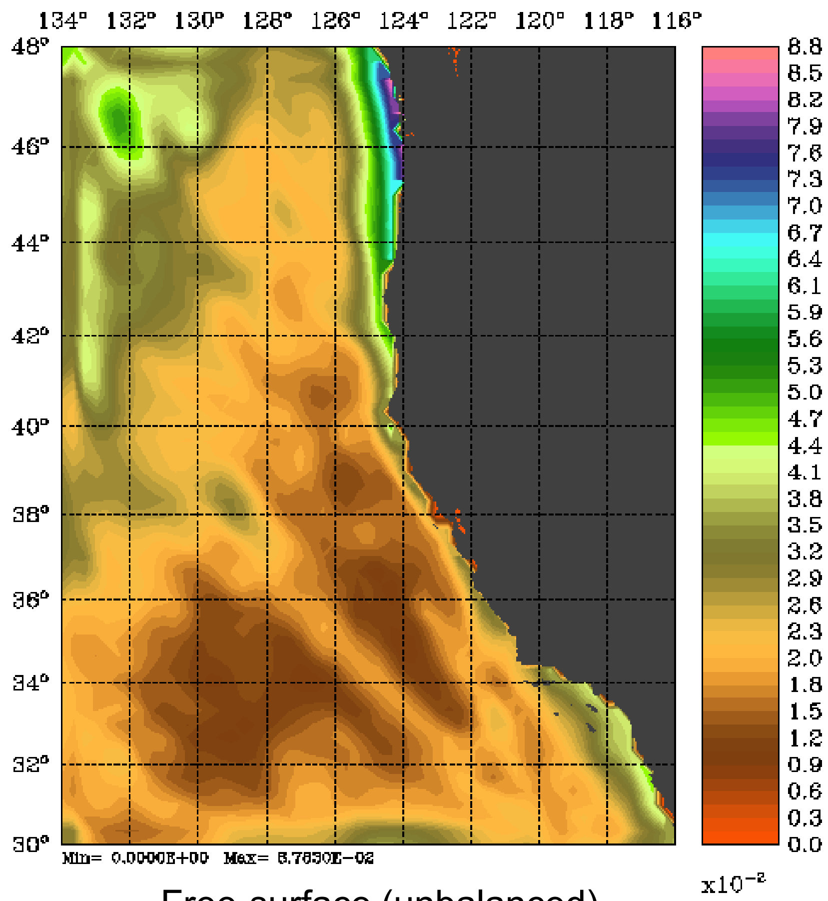
```
%  Set standard deviation NetCDF file. The file name is edited and the
%  month will be appended as *i_jan.nc:

STDfile = 'wc13_std_f.nc';

mstr = {'jan', 'feb', 'mar', 'apr', 'may', 'jun', ...
        'jul', 'aug', 'sep', 'oct', 'nov', 'dec'};

%  Set input grif file.

my_root = '/home/arango/ocean/toms/repository/test';

GRDfile = fullfile(my_root, 'WC13/Data', 'wc13_grd.nc');

%  Set input history files (string cell structure).

my_root = '/home/arango/ocean/toms/repository/test';

HISdir  = fullfile(my_root, 'WC13/STD/Data');

HISfile = dir(fullfile(HISdir, 'wc*.nc'));

nfiles = length(HISfile);

%-----------------------------------------------------------------------------
%  Compute monthly averages and standard deviations.
%-----------------------------------------------------------------------------

%  Extract first history file name.

HisFile1 = fullfile(HISdir, HISfile(1).name);
```

CCS Standard Deviation (January)

Initial Conditions



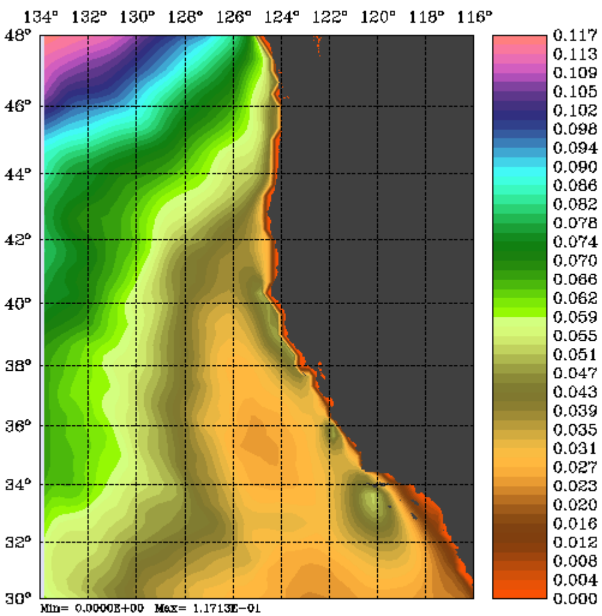
Free-surface



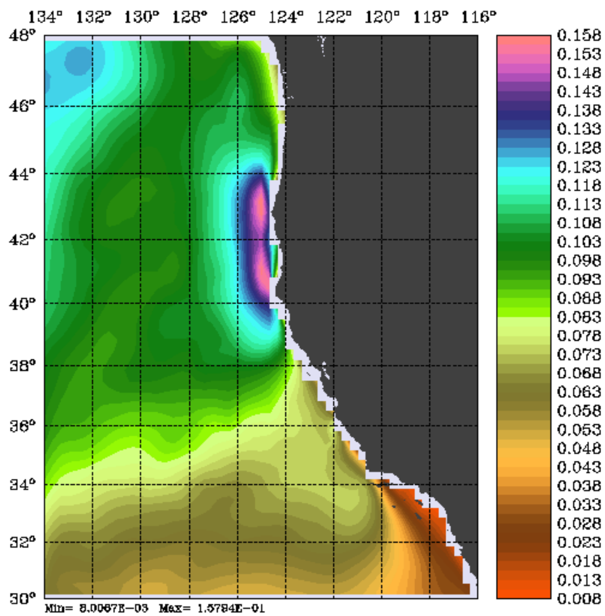
Free-surface (unbalanced)

CCS Standard Deviation (January)

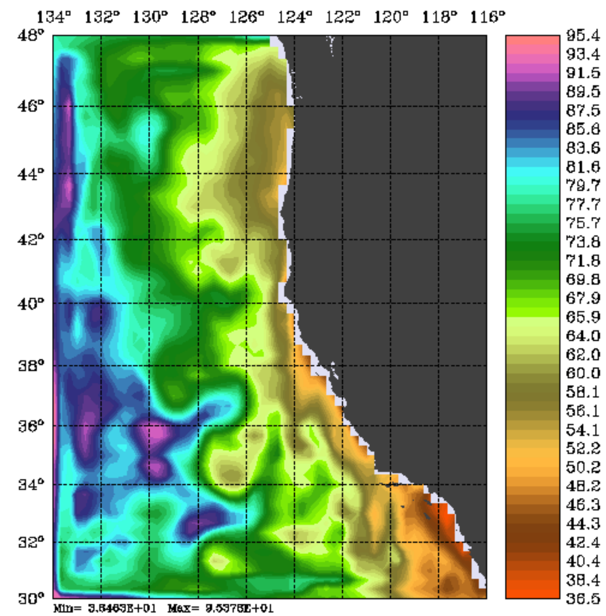
Surface Forcing



T_x



T_y

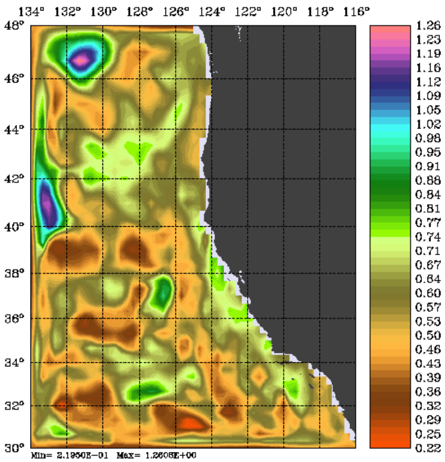


Net Heat Flux

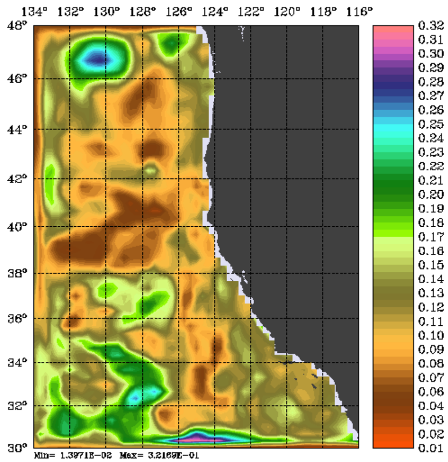
CCS Standard Deviation (January)

Initial Conditions

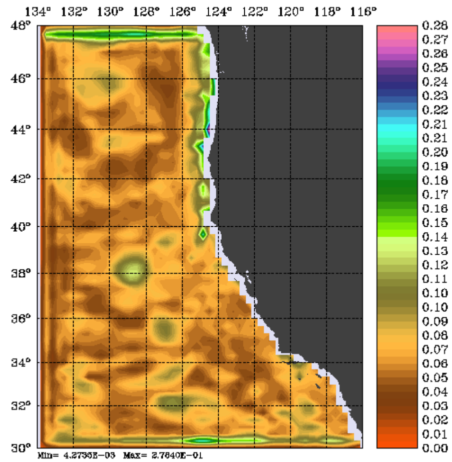
Balanced



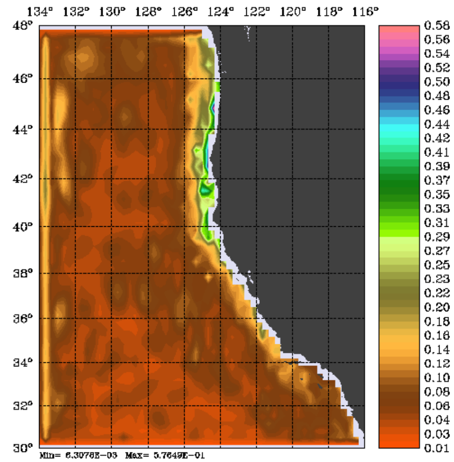
Temperature



Salinity

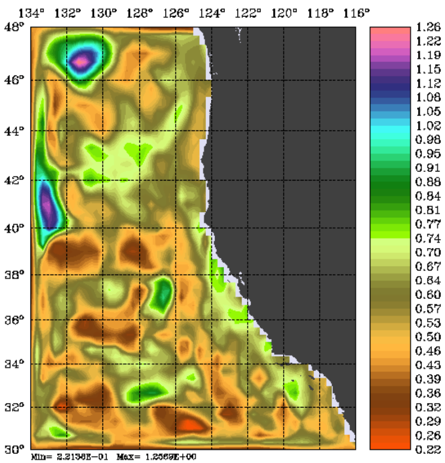


3D U-Momentum

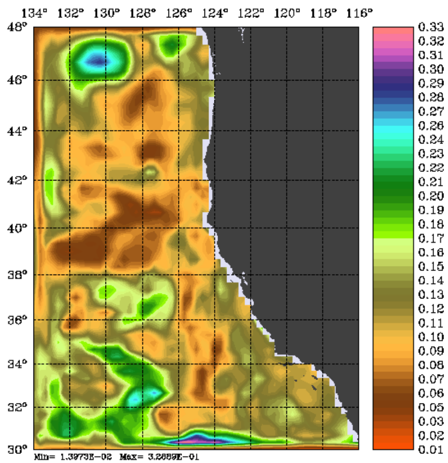


3D V-Momentum

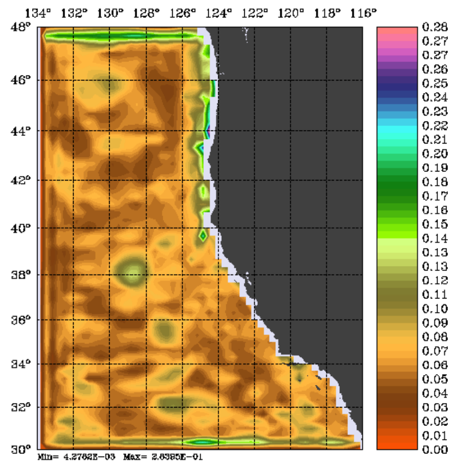
Unbalanced



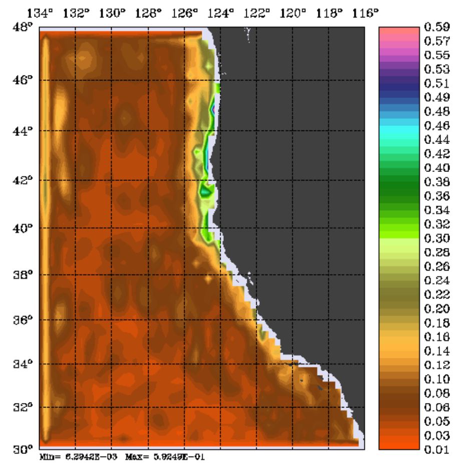
Temperature



Salinity



3D U-Momentum

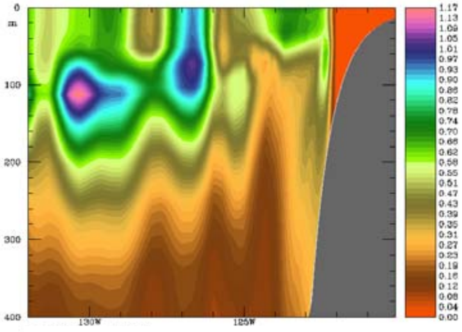


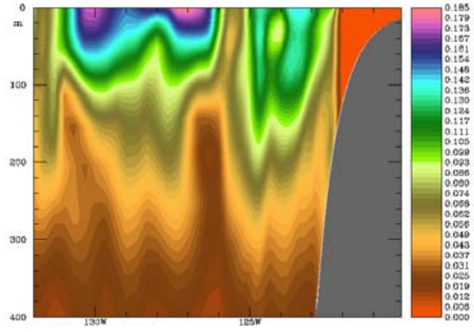
3D V-Momentum

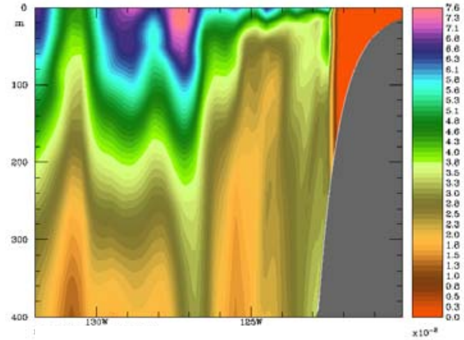
CCS Standard Deviations (January)

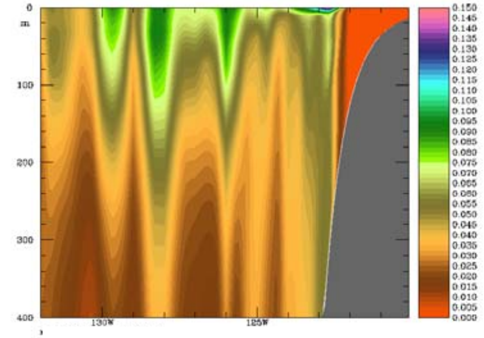
Initial Conditions, along 37°N

Balanced

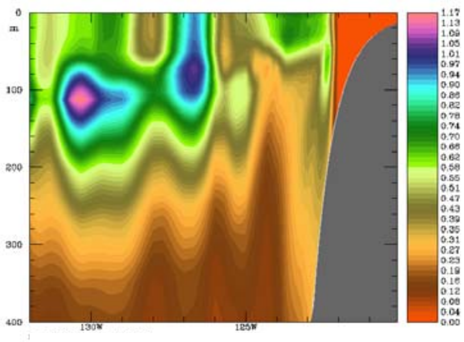

Temperature

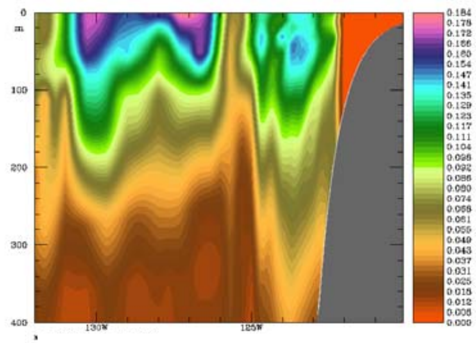

Salinity

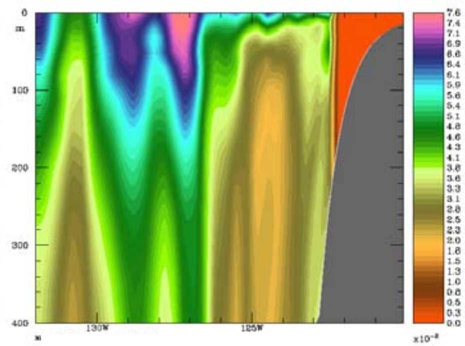

3D U-Momentum

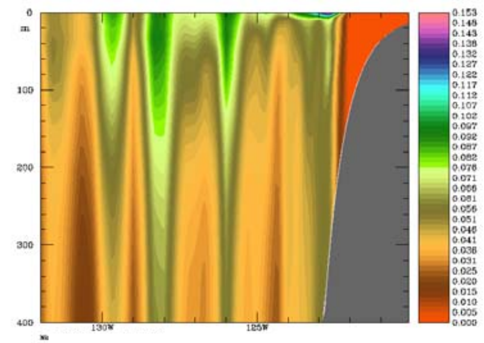

3D V-Momentum

Unbalanced


Temperature


Salinity


3D U-Momentum


3D V-Momentum

References

- Parrish, D. F., Derber, J. C., 1992: The National Meteorological Center's Spectral Statistical-Interpolation Analysis System. *Mon. Wea. Rev*., **120**, 1747-1763.
- Powell, B.S., H.G. Arango, A.M. Moore, E. DiLorenzo, R.F. Milliff and D. Foley, 2008: 4DVAR Data Assimilation in the Intra-Americas Sea with the Regional Ocean Modeling System (ROMS). *Ocean Modelling*, **23**, 130-145.