Tutorial 10:
Observation Impact &
Observation Sensitivity

Impact/Sensitivity Functions

We will use as an example, the time-averaged transport along a line of constant latitude, ϕ:

$$I = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \int_{\lambda_1}^{\lambda_2} \int_{z_1}^{z_2} v(\lambda, \phi, z)\, dz\, d\lambda\, dt$$

$$\equiv \frac{1}{(k_2 - k_1)} \sum_{k=k_1}^{k_2} \mathbf{h}_k^{\mathrm{T}} \mathbf{x}_k$$

where $[t_1, t_2] \equiv [k_1 \Delta t, k_2 \Delta t]$ **and** $\mathbf{x}_k \equiv \mathbf{x}(k\Delta t)$

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{T}_k \\ \mathbf{S}_k \\ \varsigma_k \\ \mathbf{u}_k \\ \mathbf{v}_k \end{bmatrix} \qquad \mathbf{h}_k = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \Delta z \Delta \lambda \end{bmatrix} \qquad \frac{\partial I}{\partial \mathbf{x}_j} = \frac{1}{(k_2 - k_1)} \mathbf{h}_j$$

Impact/Sensitivity Functions

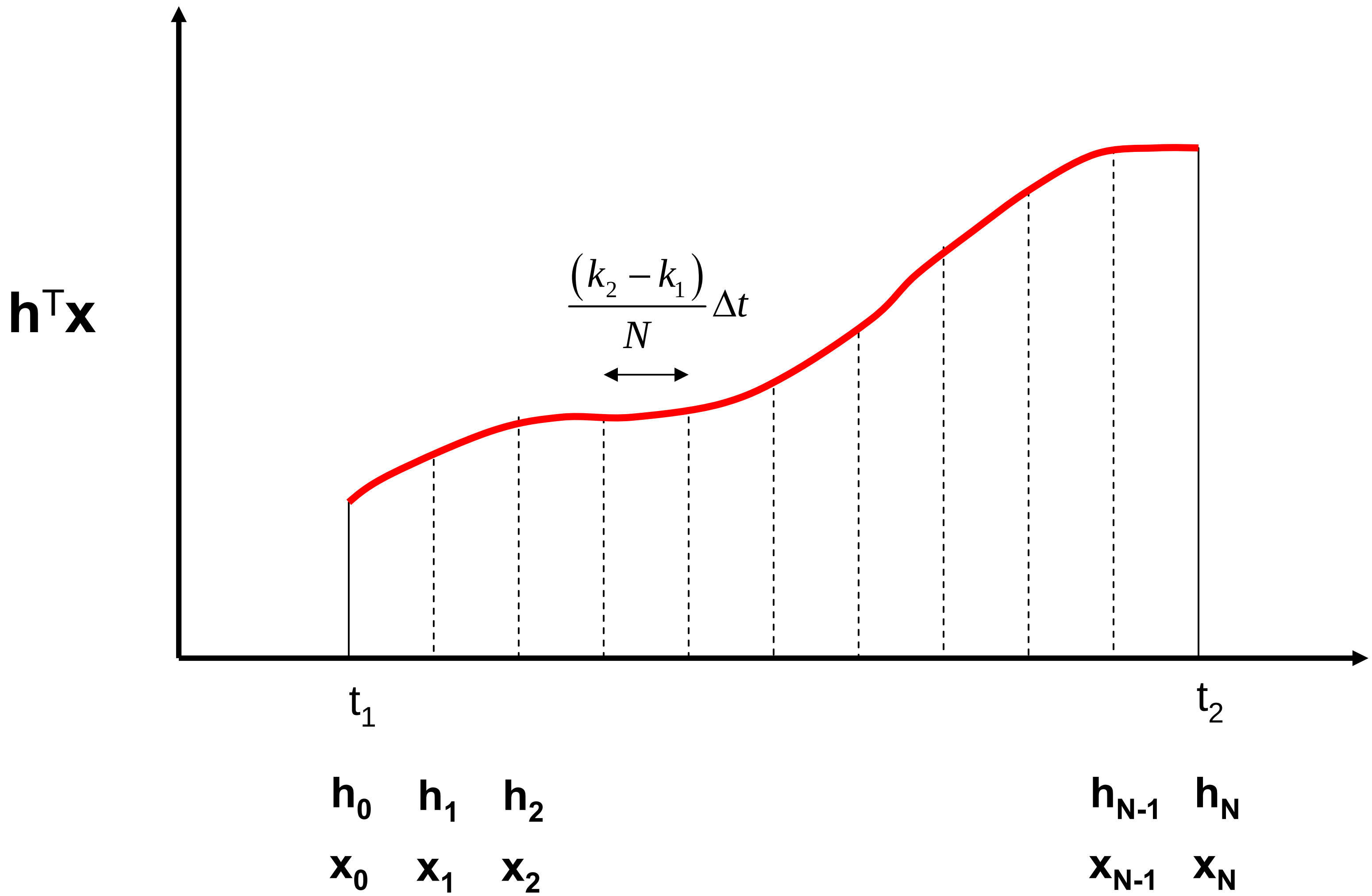
Recall that observation impact and observation sensitivity calculations involve *forcing* ADROMS with ∂*I*/∂x.

$$I = \frac{1}{(k_2 - k_1)} \sum_{k=k_1}^{k_2} \mathbf{h}_k^{\mathrm{T}} \mathbf{x}_k$$

$$\frac{\partial I}{\partial \mathbf{x}_j} = \frac{1}{(k_2 - k_1)} \mathbf{h}_j$$

The Trapezoidal Rule

Saving x every timestep is costly, so instead use an approx.

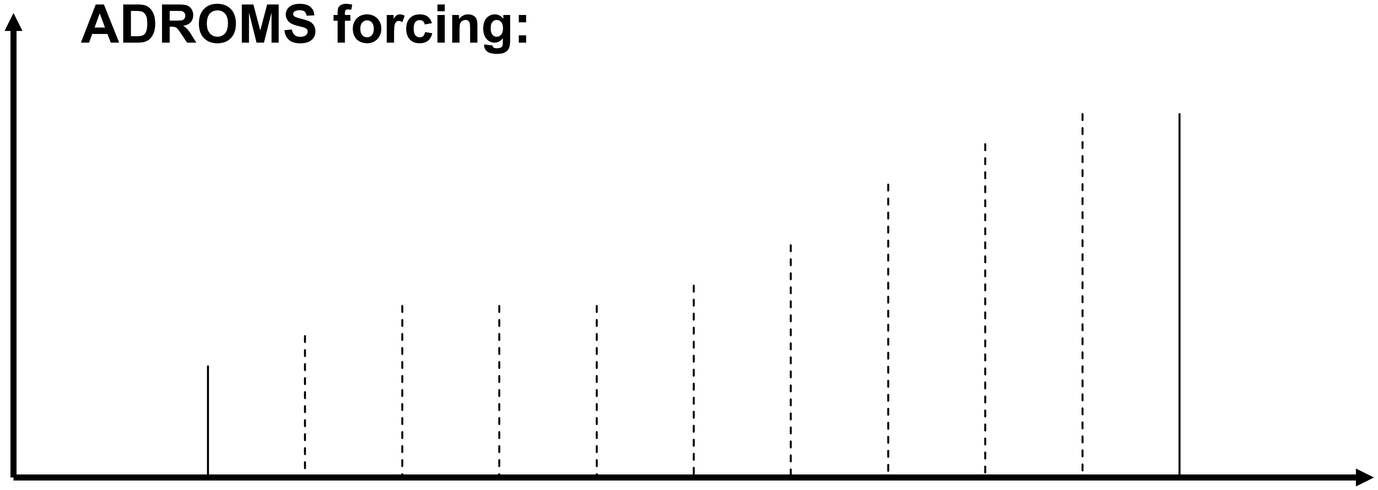


The Trapezoidal Rule

$$I = \frac{1}{N}\left(\frac{1}{2}\left(\mathbf{h}_0^{\mathrm{T}}\mathbf{x}_0 + \mathbf{h}_N^{\mathrm{T}}\mathbf{x}_N\right) + \sum_{i=1}^{N-1}\mathbf{h}_i^{\mathrm{T}}\mathbf{x}_i\right)$$

$$\frac{\partial I}{\partial \mathbf{x}_0} = \frac{1}{2N}\mathbf{h}_0; \quad \frac{\partial I}{\partial \mathbf{x}_N} = \frac{1}{2N}\mathbf{h}_N; \quad \frac{\partial I}{\partial \mathbf{x}_i} = \frac{1}{N}\mathbf{h}_i;$$

ADROMS forcing:



$\frac{\mathbf{h}_0}{2N}$ $\frac{\mathbf{h}_1}{N}$ $\frac{\mathbf{h}_2}{N}$ $\frac{\mathbf{h}_{N-1}}{N}$ $\frac{\mathbf{h}_N}{2N}$

define AD_IMPULSE

Some Common General Cases

(i) Linear *I*:

$$I = \sum_{k=k_1}^{k_2} \mathbf{h}_k^{\mathrm{T}} \mathbf{x}_k; \quad \partial \mathrm{I}/\partial \mathbf{x}_k = \mathbf{h}_k$$

(i) Quadratic *I*:

$$I = \sum_{k=k_1}^{k_2} \left(\mathbf{x}_k \pm \mathbf{g}\right)^{\mathrm{T}} \mathbf{L} \left(\mathbf{x}_k \pm \mathbf{g}\right); \quad \partial \mathrm{I}/\partial \mathbf{x}_k = \left(\mathbf{L} + \mathbf{L}^{\mathrm{T}}\right)\left(\mathbf{x}_k \pm \mathbf{g}\right)$$

where $\mathbf{g}$ is an arbitrary vector that is not a function of x.