<u>**EXERCISE 4: Weak Constraint Dual Formulation 4D-Var**</u>

Introduction

A considerable advantage of the dual form of 4D-Var over the primal form is that the dimension of the minimization problem is unaffected by the size of the control vector. Therefore, dual 4D-Var is the obvious choice for weak constraint data assimilation where the control vector is augmented with corrections for model error at every gridpoint and potentially every model time step. This exercise is a repeat of Exercise 3 except now you will run either R4D-Var or 4D-PSAS subject to weak constraint.

In some recent calculations, Broquet et al. (2010) have found that ROMS CCS tends to under estimate SST near the California coast during the peak of the upwelling season. Evidence suggests that this is associated with errors in the model rather than errors in the surface forcing. Using strong constraint 4D-Var, Broquet et al. (2010) found that data assimilation tends to compensate for model error by adjusting the surface forcing. This is clearly undesirable, and a more appropriate approach to the problem is to account for model error during the data assimilation procedure.

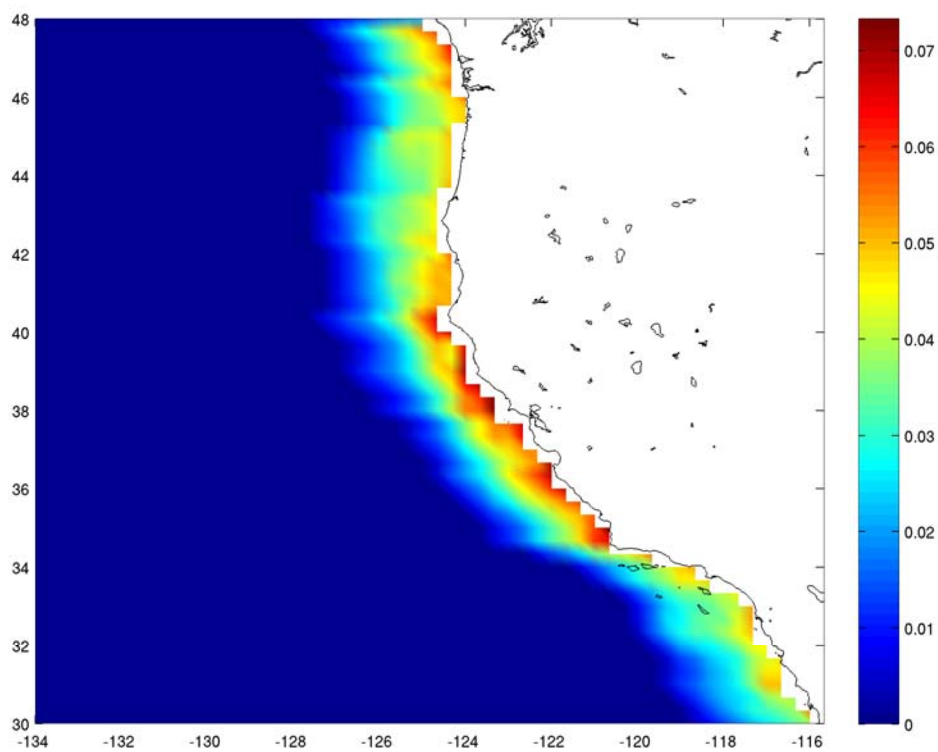

Figure 1: Assumed standard deviation for model error in the upper level of ROMS CCS.

Based on the findings of Broquet et al. (2010), a crude first attempt is made here to account for errors in the model by assuming a *prior* model error covariance of the form $\mathbf{Q} = \mathbf{K_b W \Sigma_x C \Sigma_x^T W^T K_b^T}$, where $\mathbf{\Sigma_x}$ is the *prior* initial condition error standard deviation, and **W** is a diagonal matrix with all elements zero except those that correspond to temperature gridpoints within 300 km of the coast. The non-zero elements *w* of **W** decrease linearly from a value 0.05 at the coast to zero 300 km offshore. During weak constraint 4D-Var, a correction η is applied to the model temperature at every grid point and time step, so this choice of **Q** corresponds to the situation where the standard deviation (std) of η is 5% of the std of errors in the temperature initial condition at the coast, decreasing linearly to zero offshore. The structure of $\mathbf{W\Sigma_x}$ corresponding to the surface level temperature is illustrated in Fig. 1.

Running weak constraint R4D-Var and 4D-PSAS

For this exercise run the same dual 4D-Var algorithm as you used for Exercise 3 (i.e. either R4D-Var or 4D-PSAS).

If you have chosen to run R4D-Var, the indirect representer approach to the dual form, then go to the directory **WC13/R4DVAR.**

If you have chosen to run 4D-PSAS, the physical-space statistical analysis approach to the dual form, then go to the directory **WC13/PSAS.**

VERY IMPORTANT: Before running Exercise 4, create a new subdirectory called **EX3** and move all of the output files (*.nc) generated by Exercise 3 (including the log file) to that directory, otherwise they will be overwritten.

In either case, follow the instructions in the appropriate **Readme** file. Before running the model, you will need to edit **ocean_wc13.in** to indicate that the weak constraint will be used. This is controlled by the parameter *NADJ*. If *NADJ*<*NTIMES*, then the weak constraint is assumed, otherwise dual 4D-Var is run using the constraint. Choosing *NADJ*<*NTIMES* causes *NADJ* to be reset to the value of *NHIS*, which controls the frequency at which the model error corrections η. The resulting corrections are linearly interpolated in time to yield a correction at every time step, a procedure that necessarily introduces a temporal correlation in the model error corrections. Choosing *NHIS*=1 but is not advisable due to the large amount of I/O that will be involved.

Plotting your results

Plot the 4D-Var cost function *J* and its components $J_{b,}$ and J_o using **plot_r4dvar_cost.m** or **plot_psas_cost.m** as appropriate, which can both be found in **WC13/plotting**.

Plot next a selection of the increments using **plot_r4dvar_increments.m** or **plot_psas_increments.m**, as appropriate.

NOTE: Before moving on to Exercise 5, please move all of the *.nc files and the log file into a new directory.

References

Broquet, G., A.M. Moore, H.G. Arango, and C.A. Edwards, 2010: Corrections to ocean surface forcing in the California Current System using 4D-variational data assimilation. *Ocean Modelling*, Under review.