EXERCISE 3: Dual Formulation 4D-Var: R4D-Var and 4D-PSAS

Introduction

As discussed in Lecture 3, 4D-Var can cast in a dual form in which the search for the best ocean circulation estimate proceeds in the subspace spanned only by the observations, as opposed to the full space spanned by the model (i.e. the primal form and I4D-Var of Exercises 1 and 2). Formally, the primal and dual formulations yield identical estimates of the ocean circulation so one might wonder if there is any advantage of one form over the other? In this exercise and Exercise 4, we will explore this question, and discover that there are indeed practical advantages and disadvantages to both approaches.

Running R4D-Var and 4D-PSAS

First of all, choose which of the dual formulations you would like to use for this exercise. There is only time to run one of them during the workshop, but you can return to this exercise at your leisure and run the second case if you are so inclined.

If you have chosen to run R4D-Var, the indirect representer approach to the dual form, then go to the directory **WC13/R4DVAR.**

If you have chosen to run 4D-PSAS, the physical-space statistical analysis approach to the dual form, then go to the directory **WC13/PSAS.**

In either case, follow the instructions in the appropriate **Readme** file, and run the model. As in Exercise 1, **ocean_wc13.in** is configured to perform one outer-loop and 50 inner-loops. However, if you used *Ninner<50* for Exercise 1, then use the same number of inner-loops here as you used for Exercise 1. Using the same combination of *Nouter* and *Ninner* for Exercises 1 and 3 will greatly facilitate the comparisons of the primal and dual circulation estimates later.

Plotting your results

1. Plot the 4D-Var cost function J and its components $J_{b,}$ and J_o using **plot_r4dvar_cost.m** or **plot_psas_cost.m** as appropriate, which can both be found in **WC13/plotting**.
2. Plot next a selection of the increments using **plot_r4dvar_increments.m** or **plot_psas_increments.m**, as appropriate.
3. As described during Lecture 4, the dual forms of ROMS 4D-Var also provide estimates of the expected *posterior*/analysis error variance. A selection of plots of the expected *posterior* error variance compared to the *prior* error variance can be generated using **plot_prior_posterior_errors.m**. Before running this script you will need to edit the pathname at line 25 so that *Inpa* points to the appropriate directory depending on whether you ran R4D-Var or 4D-PSAS.
4. In addition, the dual forms of ROMS 4D-Var will compute estimates of the leading eigenvectors (Empirical Orthogonal Functions, EOFs) of the expected

posterior/analysis error covariance matrix $\mathbf{E^a}$. The script **plot_eof_eigenvalues.m** will plot the eigenvalues λ_i of $\mathbf{E^a}$ and an estimate of the uncertainty $\varepsilon_i = \lambda_1^{-1}\left\|\mathbf{E^a}\hat{\mathbf{z}}_i - \hat{\mathbf{z}}_i\right\| / \left\|\hat{\mathbf{z}}_i\right\|$ associated with each EOFs $\hat{\mathbf{z}}_i$. Also shown are estimates of $\mathrm{Tr}\{\mathbf{E^a}\}$ generated using a randomized trace estimate technique, where $\lambda_i / \mathrm{Tr}\{\mathbf{E^a}\}$ represents the fraction of variance explained by EOF $\hat{\mathbf{z}}_i$. As in (3) you will need to edit the pathname at line 37 for *Inp*.

5. Finally, you can plot some of the fields associated with a particular EOF of $\mathbf{E^a}$ using **plot_posterior_EOF_fields.m**. Edit this script to select the EOF you would like to plot via *nEOF* at line 16, and edit the pathname at line 25 as appropriate.