EXERCISE 2: I4D-Var with Multiple Outer-loops

Introduction

It is often advantageous to run 4D-Var using multiple outer-loops. During each outer-loop the circulation about which the tangent linear and adjoint models are linearized is updated by rerunning the nonlinear model using the increments from the end of the previous outer-loop. Using this approach 4D-Var proceeds as a sequence of linear least-squares minimizations, which (hopefully) approximate minimization of the nonlinear cost function:

$$J_{NL} = \frac{1}{2}\left(\mathbf{z}-\mathbf{z_b}\right)^{\mathrm{T}}\mathbf{D}^{-1}\left(\mathbf{z}-\mathbf{z_b}\right)+\frac{1}{2}\left(\mathbf{y}-H(\mathbf{x})\right)\mathbf{R}^{-1}\left(\mathbf{y}-H(\mathbf{x})\right)$$

Running I4D-Var with multiple outer-loops

This exercise is essentially a repeat of Exercise 1, except that you will use a different number of outer-loops.

VERY IMPORTANT: Before running Exercise 2, create a new subdirectory called **EX1**, and move all of the output files (*.nc) generated by Exercise 1 (including the log file) to that directory, otherwise they will be overwritten.

To run this exercise, go first to the directory **WC13/I4DVAR**. The only change that you need to make is to **ocean_wc13.in**, where you need to select new values for *Ninner* and *Nouter*. If you ran Exercise 1 using *Ninner*=50, then choose **one** of the following combinations of *Nouter* and *Ninner* which yield approximately the same total number of iterations *Nouter*×*Ninner*≈50.

- *Nouter*=2, *Ninner*=25
- *Nouter*=3, *Ninner*=17
- *Nouter*=4, *Ninner*=13
- *Nouter*=5, *Ninner*=10

Or feel free to choose another combination of *Nouter* and *Ninner* that yields ~50 iterations total.

If you ran Exercise 1 using *Ninner*<50, then choose a combination of *Nouter* and *Ninner* that yields a similar total number of iterations for your case.

Now go ahead and run I4D-Var as described in the **Readme** file.

Plotting your results

Plot the I4D-Var cost function J and its components J_b, J_o and the theoretical minimum value $J_{min}=N_{obs}/2$ using **WC13/plotting/plot_i4dvar_cost.m**.