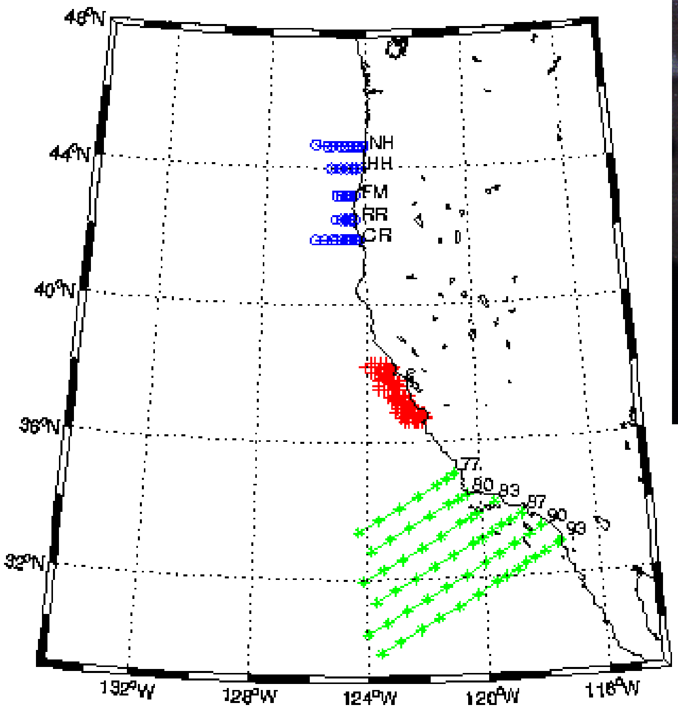
Lecture 5:
Observation Impact & Observation Sensitivity

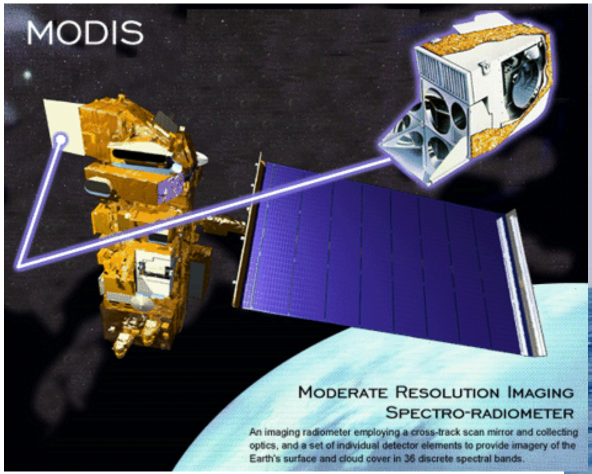
Outline

- Observation impacts
- Adjoint 4D-Var: $(\text{4D-Var})^T$
- Observation sensitivity
- Error covariance estimates from $(\text{4D-Var})^T$

Observation Impacts

(Useful references: Langland & Baker, 2004;
Gelaro and Zhu, 2009; Tremolet, 2008)





An imaging radiometer employing a cross-track scan mirror and collecting optics, and a set of individual detector elements to provide imagery of the Earth's surface and cloud cover in 36 discrete spectral bands.

Photo Dan Costa

Given the plethora of different observation platforms, what impact does each have on the 4D-Var analysis?

Observation Impacts

Consider a scalar function of the ocean state vector:

$$I = I(\mathbf{x})$$

Prior **Posterior** **Increment**

$$I_{\mathrm{b}} = I(\mathbf{x}_{\mathrm{b}}) \qquad I_{\mathrm{a}} = I(\mathbf{x}_{\mathrm{a}}) \qquad \Delta I = I_{\mathrm{a}} - I_{\mathrm{b}}$$

$$\mathbf{x}_{\mathrm{a}}(\mathrm{t}) = \mathbf{x}_{\mathrm{b}}(\mathrm{t}) + \delta\mathbf{x}(\mathrm{t})$$

$$I_{\mathrm{a}} = I(\mathbf{x}_{\mathrm{b}} + \delta\mathbf{x}) \simeq I_{\mathrm{b}} + \delta\mathbf{x}^{\mathrm{T}} * (\partial I/\partial\mathbf{x})$$

$$\Delta I \simeq \delta\mathbf{x}^{\mathrm{T}} * (\partial I/\partial\mathbf{x}) \text{ **but** } \delta\mathbf{x}(t) = \mathbf{M}_{\mathrm{b}}(t, t_0)\tilde{\mathbf{K}}\mathbf{d}$$

$$\boxed{\Delta I \simeq \mathbf{d}^{\mathrm{T}}\tilde{\mathbf{K}}^{\mathrm{T}}\mathbf{M}_{\mathrm{b}}^{\mathrm{T}} * (\partial I/\partial\mathbf{x})}$$

($\mathbf{M}_{\mathrm{b}}^{\mathrm{T}} * (\partial I/\partial\mathbf{x})$ denotes a time convolution)

Observation Impacts

Consider a scalar function of the ocean state vector:

$$I = I(\mathbf{x})$$

Prior

$$I_{\mathrm{b}} = I(\mathbf{x}_{\mathrm{b}})$$

Posterior

$$I_{\mathrm{a}} = I(\mathbf{x}_{\mathrm{a}})$$

Increment

$$\Delta I = I_{\mathrm{a}} - I_{\mathrm{b}}$$

$$\boxed{\Delta I \simeq \underset{\substack{\uparrow \\ \text{Innovations}}}{\mathbf{d}^{\mathrm{T}}} \underset{\substack{\uparrow \\ \text{Adjoint of} \\ \text{gain matrix}}}{\tilde{\mathbf{K}}^{\mathrm{T}}} \underset{\substack{\uparrow \\ \text{Adjoint model}}}{\mathbf{M}^{\mathrm{T}}} * (\partial I / \partial \mathbf{x})}$$

Observation Impacts

Recall the dual form of the gain matrix:

$$\mathbf{K} \approx \tilde{\mathbf{K}}_k = \mathbf{D}\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1/2}\mathbf{V}_k\mathbf{T}_k^{-1}\mathbf{V}_k^{\mathrm{T}}\mathbf{R}^{-1/2}$$

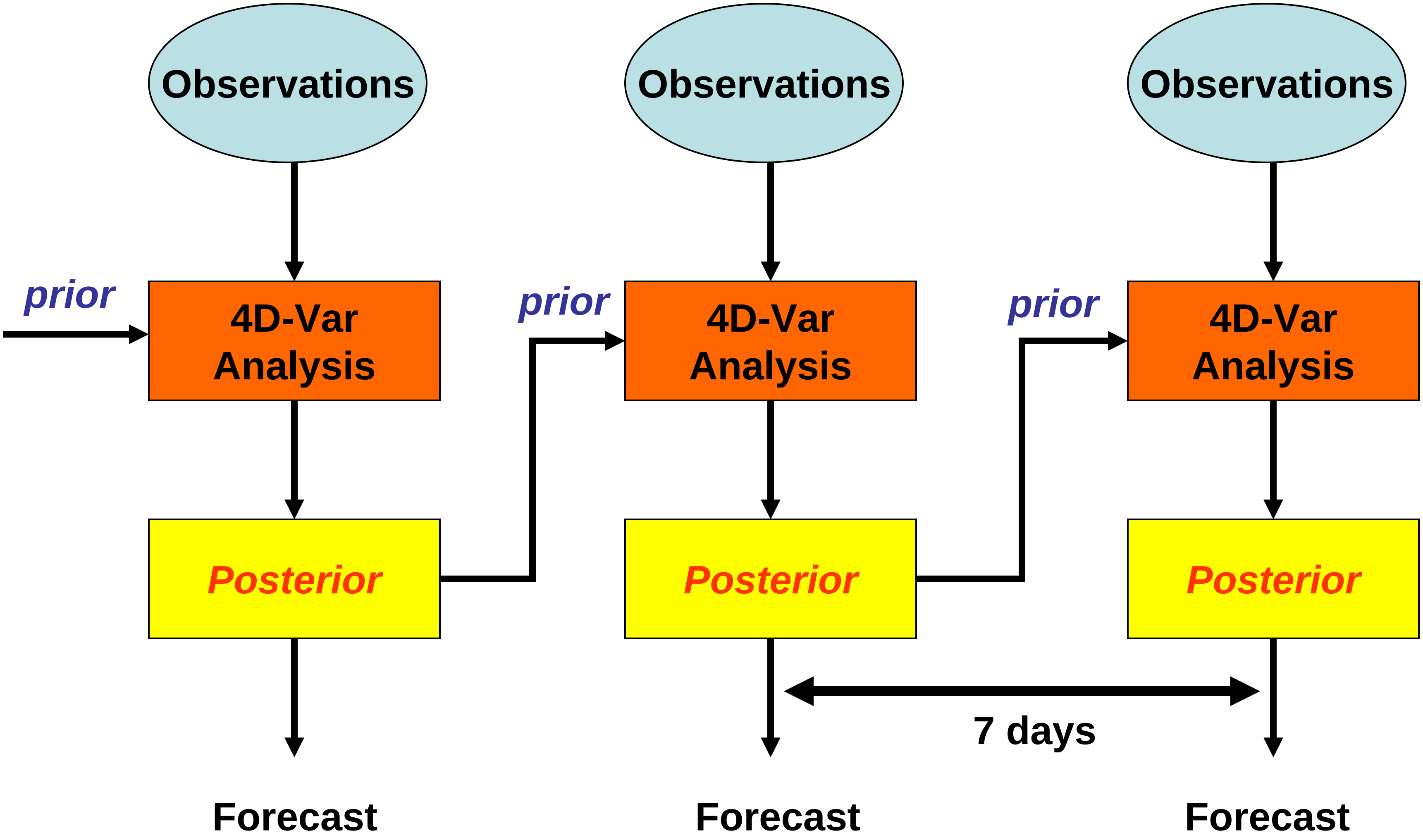
So:

$$\tilde{\mathbf{K}}_k^{\mathrm{T}} = \mathbf{R}^{-1/2}\mathbf{V}_k\mathbf{T}_k^{-1}\mathbf{V}_k^{\mathrm{T}}\mathbf{R}^{-1/2}\mathbf{G}\mathbf{D}$$

Therefore:

$$\Delta I \simeq \mathbf{d}^{\mathrm{T}}\mathbf{R}^{-1/2}\mathbf{V}_k\mathbf{T}_k^{-1}\mathbf{V}_k^{\mathrm{T}}\mathbf{R}^{-1/2}\mathbf{G}\mathbf{D}\mathbf{M}^{\mathrm{T}} * (\partial I / \partial \mathbf{x})$$

Sequential 4D-Var with 10km CCS ROMS



Observations

Observations

Observations

prior

4D-Var Analysis

prior

4D-Var Analysis

prior

4D-Var Analysis

Posterior

Posterior

Posterior

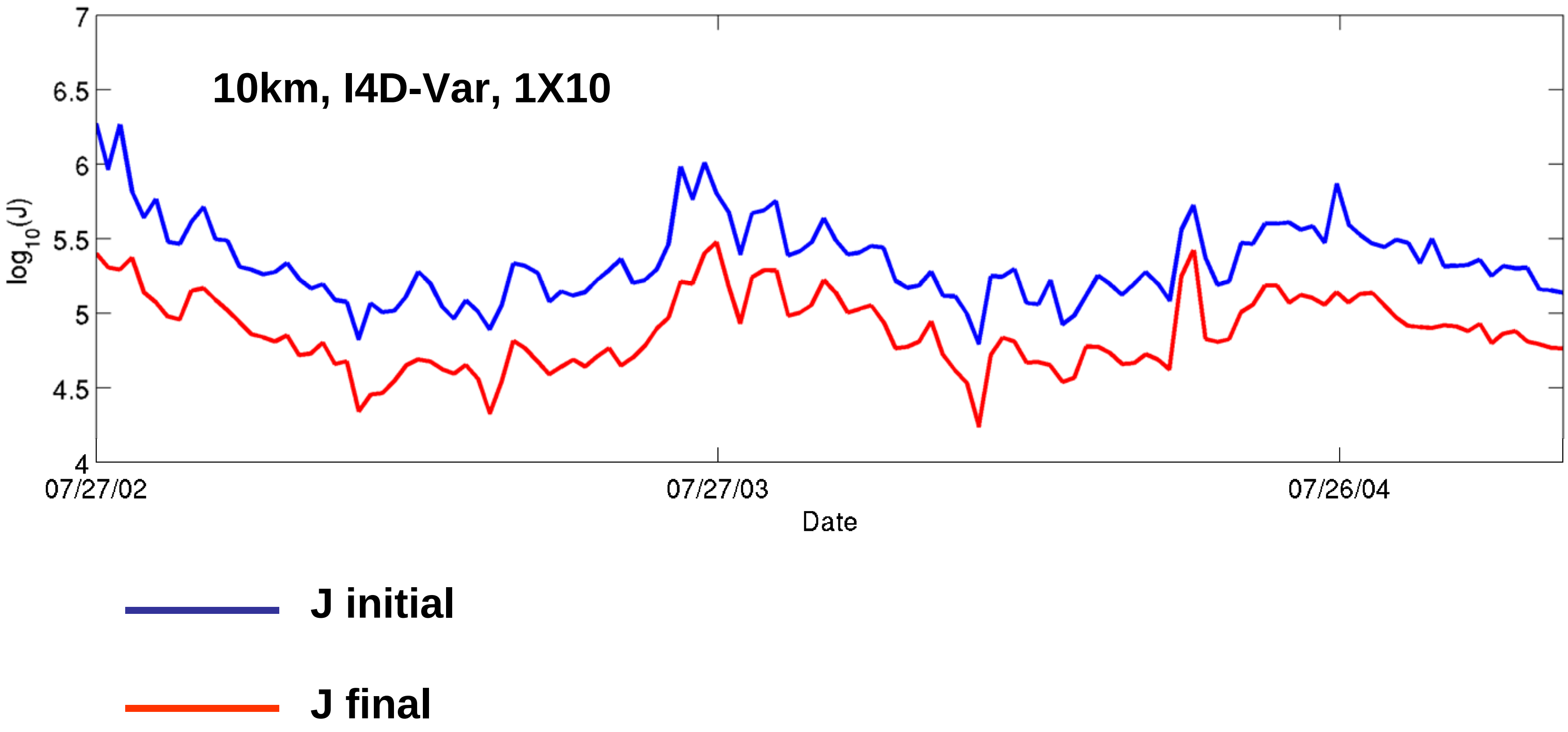
7 days

Forecast

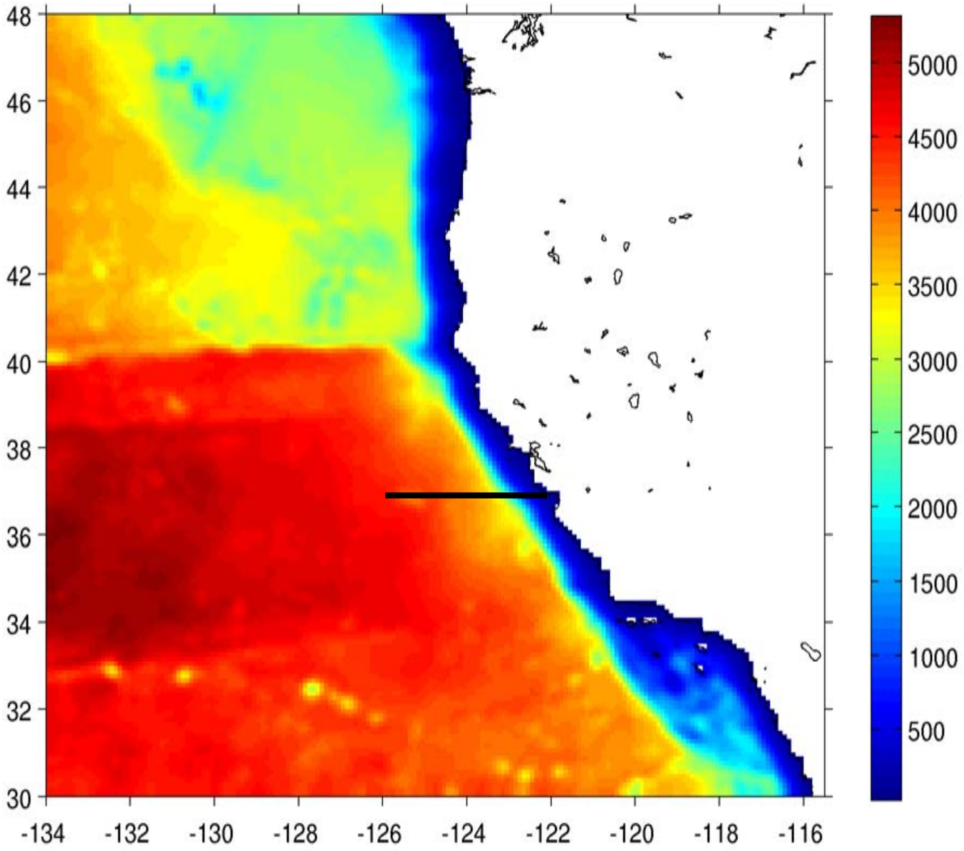
Forecast

Forecast

Sequential 4D-Var CCS ROMS



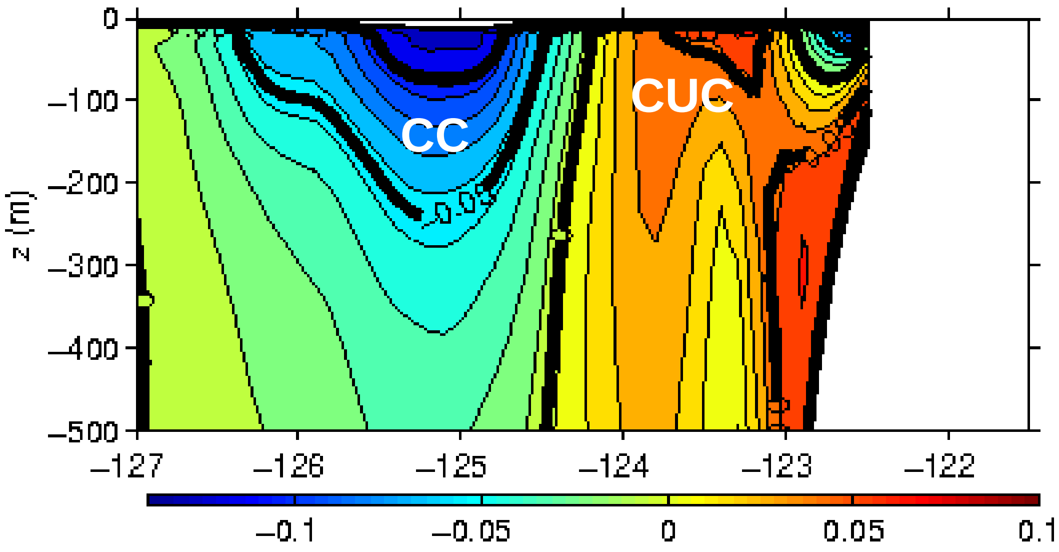
Example: 37N Transport



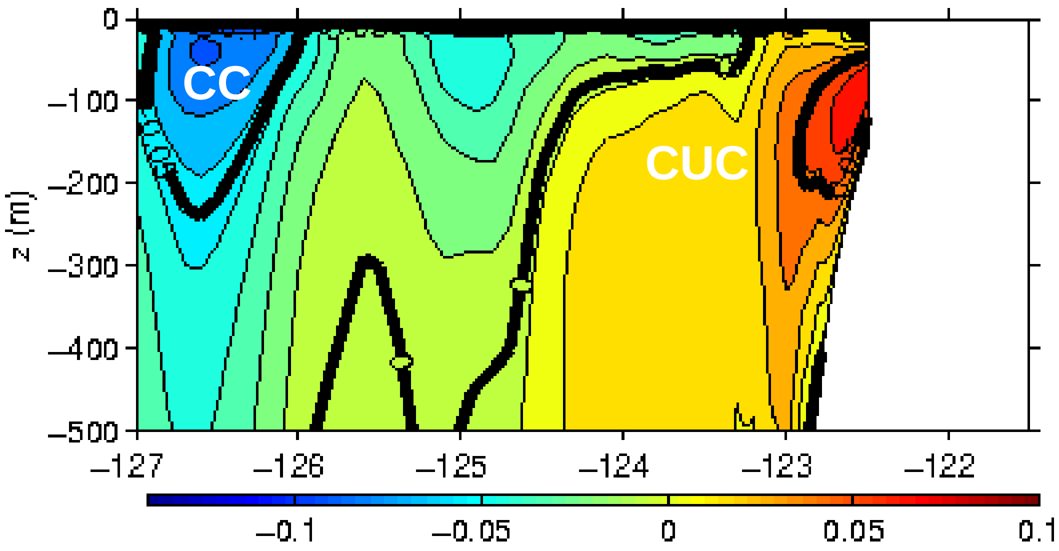
10km, CCS ROMS

Example: 37N Transport

No assim



Primal Strong



JAS time mean alongshore Flow (10km, 42 lev)

CC = California Current

CUC = California Under Current

37N Transport Observation Impacts

The time average 37N transport can be written as:

$$I_{37N} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{h}^{\mathrm{T}}\mathbf{x}_i$$

where: $\mathbf{x}_i \equiv \mathbf{x}(i\Delta t) = \mathbf{x}(t)$

↑

Model timestep

therefore:

$$\Delta I_{37N} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{h}^{\mathrm{T}}\left((\mathbf{x}_{\mathbf{a}})_i - (\mathbf{x}_{\mathbf{b}})_i\right)$$

$\mathbf{M}_{\mathbf{b}}^{\mathrm{T}} * (\partial I/\partial \mathbf{x})$

$$\simeq \frac{1}{N}\sum_{i=1}^{N}\mathbf{h}^{\mathrm{T}}(\mathbf{M}_{\mathbf{b}})_i\tilde{\mathbf{K}}\mathbf{d} = \mathbf{d}^{\mathrm{T}}\tilde{\mathbf{K}}^{\mathrm{T}}\boxed{\sum_{i=1}^{N}\frac{1}{N}(\mathbf{M}_{\mathbf{b}})_i^{\mathrm{T}}\mathbf{h}}$$

where: $(\mathbf{M}_{\mathbf{b}})_i \equiv \mathbf{M}(t_0 + i\Delta t, t_0) = \mathbf{M}(t, t_0)$

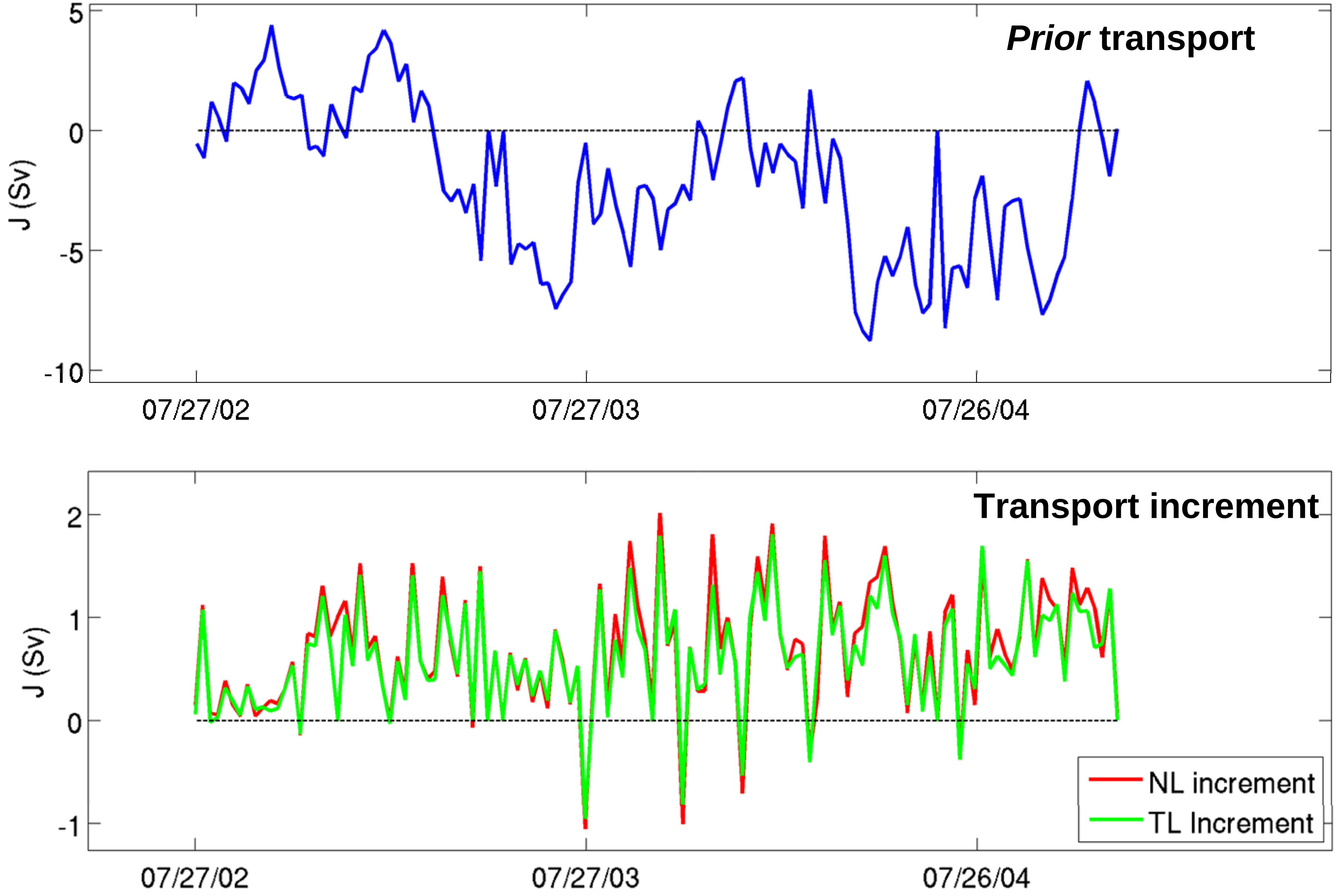
37N Transport Observation Impacts

37N time averaged transport increment:

$$\Delta I_{37N} \simeq \frac{1}{N}\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{K}}^{\mathrm{T}}\underbrace{\sum_{i=1}^{N}(\mathbf{M}_{\mathbf{b}})_i^{\mathrm{T}}\mathbf{h}}_{\text{ADROMS forced by h}}$$

$$\tilde{\mathbf{K}}_k^{\mathrm{T}} = \mathbf{R}^{-1/2}\underbrace{\mathbf{V}_k\mathbf{T}_k^{-1}\mathbf{V}_k^{\mathrm{T}}}_{\substack{\text{Dual space}\\ \text{Lanczos vectors}}}\mathbf{R}^{-1/2}\underset{\substack{\uparrow\\ \text{TLROMS sampled at}\\ \text{observation points}}}{\mathbf{G}}\mathbf{D}$$

37N Transport



Control Vector Impacts

37N time averaged transport increment:

$$\Delta I_{37N} \simeq \frac{1}{N}\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{K}}^{\mathrm{T}}\sum_{i=1}^{N}(\mathbf{M_b})_i^{\mathrm{T}}\mathbf{h}$$

$$= \mathbf{d}^{\mathrm{T}}\mathbf{g} = \mathbf{d}^{\mathrm{T}}\left(\mathbf{g_x} + \mathbf{g_f} + \mathbf{g_b}\right)$$

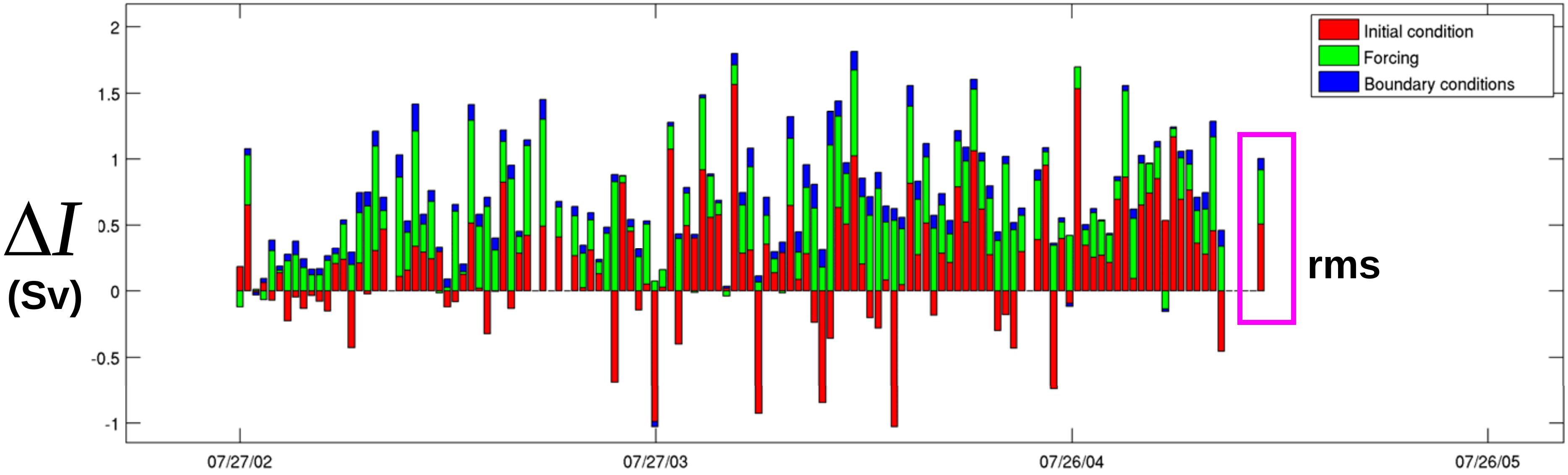
where: $\mathbf{g} \simeq \frac{1}{N}\tilde{\mathbf{K}}\sum_{i=1}^{N}(\mathbf{M_b})_i^{\mathrm{T}}\mathbf{h}$

$\mathbf{g_x}$ **- contribution from initial condition increments**

$\mathbf{g_f}$ **- contribution from surface forcing increments**

$\mathbf{g_b}$ **- contribution from open boundary increments**

37N Transport Control Vector Impacts



Observation Impacts

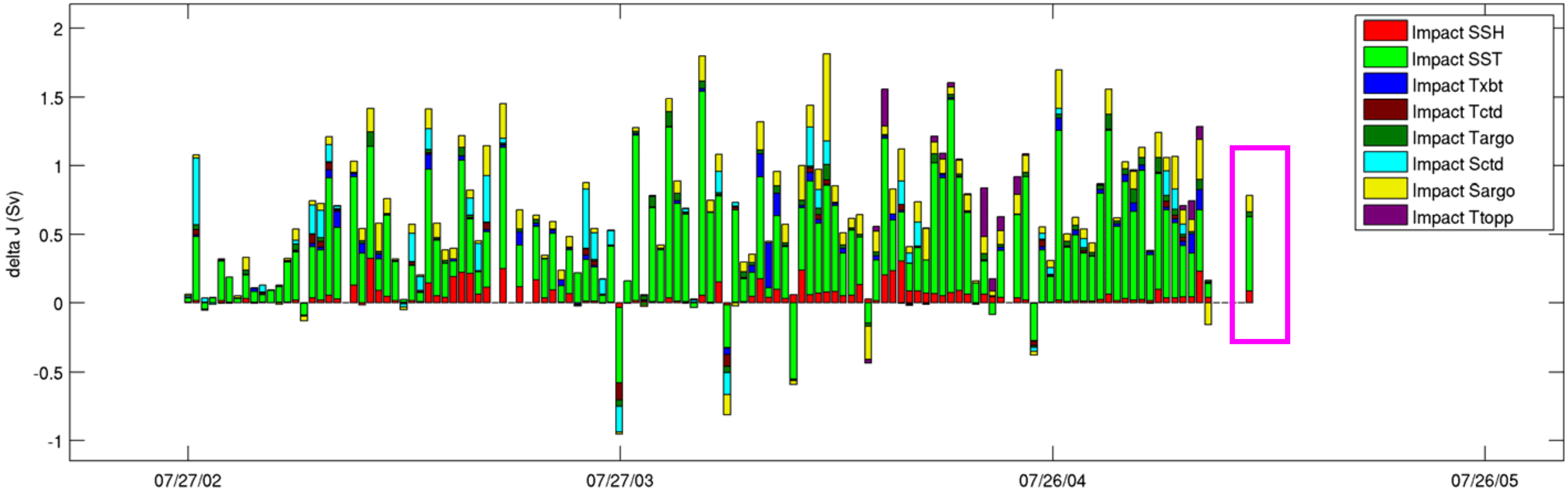
37N time averaged transport increment:

$$\Delta I_{37N} \simeq \frac{1}{N}\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{K}}^{\mathrm{T}}\sum_{i=1}^{N}(\mathbf{M_b})_i^{\mathrm{T}}\mathbf{h}$$

$$=\mathbf{d}^{\mathrm{T}}\mathbf{g}=\sum_{i=1}^{N_{obs}} d_i g_i$$

$$=\sum_{i=1}^{N_{obs}}\underbrace{\left(y_i - H_i\left(\mathbf{x_b}(t)\right)\right)}_{\text{Contribution of each observation to }\Delta\text{I}} g_i$$

37N Transport Observation Impacts



🟥 **Satellite SSH**

🟩 **Satellite SST**

🟩 **T Argo**

🟦 **T XBT**

🟫 **T CTD**

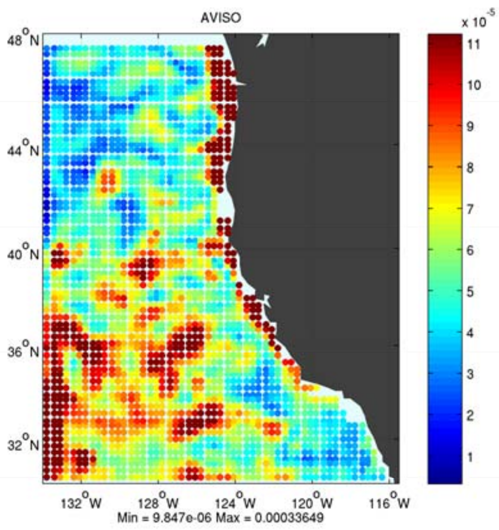
🟪 **T TOPP**

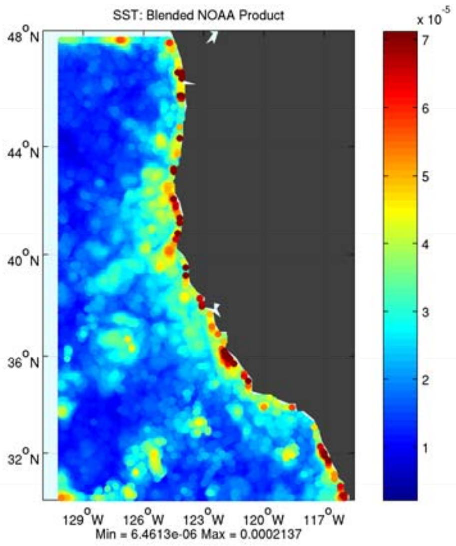
🟦 **S CTD**

🟨 **S Argo**

37N Transport Observation Impacts

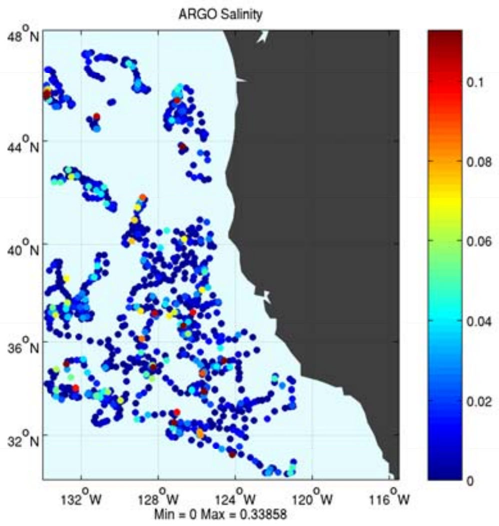
SSH

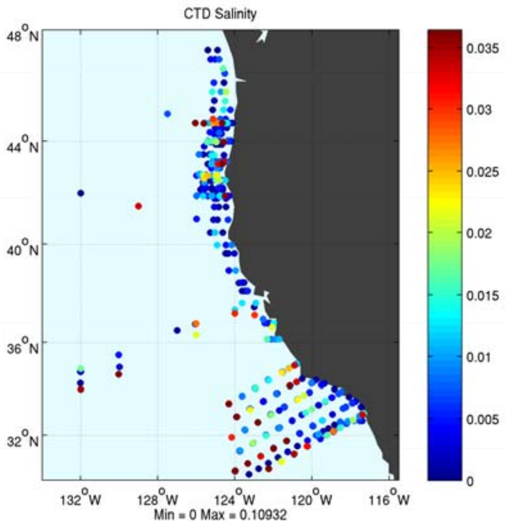




SST

Argo S





CTD S

Two Spaces: Obs Impact

$\mathbf{K}$ maps from observation (dual) space
to model (primal) space

$\mathbf{K}^{\mathrm{T}}$ maps from model (primal) space
to observation (dual) space

Identifies the part of model space that controls 37N transport
and that is activated by the observations

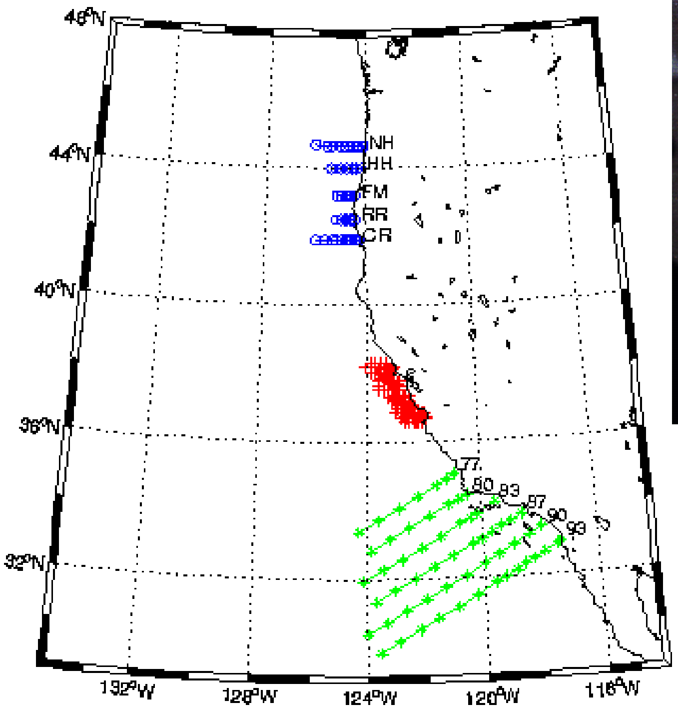
Observation Impacts: ROMS Implementation

- **Primal (I4D-Var) and dual (4D-PSAS & R4D-Var) forms available:**
  - **define IS4DVAR_SENSITIVITY**
    Drivers/obs_sen_is4dvar.h

  - **define W4DPSAS_SENSITIVITY**
    define OBS_IMPACT
    define OBS_IMPACT_SPLIT
    Drivers/obs_sen_w4dpsas.h

  - **define W4DVAR_SENSITIVITY**
    define OBS_IMPACT
    define OBS_IMPACT_SPLIT
    Drivers/obs_sen_w4dvar.h

Adjoint 4D-Var & Observation Sensitivity



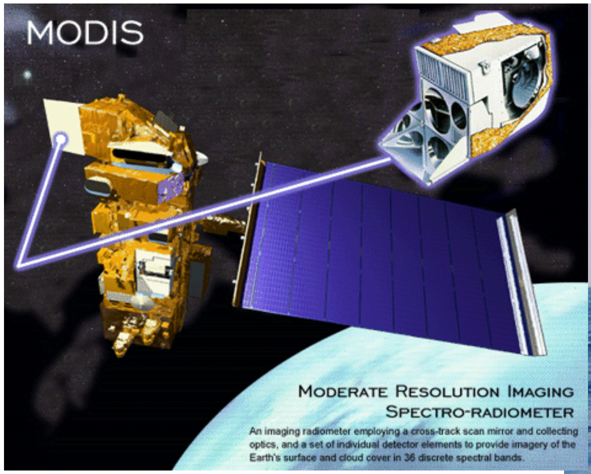



Photo Dan Costa

How will the circulation analysis change if some of the observations or the observation array change?

Adjoint 4D-Var & Observation Sensitivity

The analysis increments are a nonlinear function of the innovation vector d:

4D-Var

$$\mathbf{z}_\mathbf{a} = \mathbf{z}_\mathbf{b} + \boxed{K(\mathbf{d})}$$

where:

$$\mathbf{d} = \mathbf{y} - H\left(\mathbf{z}_\mathbf{b}(t)\right)$$

Consider variations in the observation vector δy:

$$\delta\mathbf{d} = \delta\mathbf{y}; \quad \mathbf{z}_\mathbf{a} + \delta\mathbf{z}_\mathbf{a} = \mathbf{z}_\mathbf{b} + K(\mathbf{d} + \delta\mathbf{d})$$

$$\approx \mathbf{z}_\mathbf{b} + K(\mathbf{d}) + \left(\partial K / \partial\mathbf{y}\right)\delta\mathbf{y}$$

$$\delta\mathbf{z}_\mathbf{a} \approx \boxed{\frac{\partial K}{\partial\mathbf{y}}}\delta\mathbf{y}$$

Tangent
linearization
of 4D-Var

Adjoint 4D-Var & Observation Sensitivity

Consider a scalar function of the *posterior* control vector $\mathbf{z_a}$:

$$I_{\mathbf{a}} = I(\mathbf{z_a}) = I(\mathbf{z_b} + \mathcal{K}(\mathbf{d}))$$

A change $\delta\mathbf{y}$ in the observations yields a change in $\Delta I_{\mathbf{a}}$:

$$I_{\mathbf{a}} + \Delta I_{\mathbf{a}} = I(\mathbf{z_b} + \mathcal{K}(\mathbf{d} + \delta\mathbf{y}))$$

$$\approx I\left(\mathbf{z_b} + \mathcal{K}(\mathbf{d}) + (\partial\mathcal{K}/\partial\mathbf{y})\,\delta\mathbf{y}\right)$$

$$\approx I(\mathbf{z_a}) + \left((\partial\mathcal{K}/\partial\mathbf{y})\,\delta\mathbf{y}\right)^{\mathrm{T}}\left(\partial I/\partial\mathbf{z}\right)$$

Therefore:

$$\Delta I_{\mathbf{a}} \approx \delta\mathbf{y}^{\mathrm{T}}\left(\partial\mathcal{K}/\partial\mathbf{y}\right)^{\mathrm{T}}\left(\partial I/\partial\mathbf{z}\right)$$

Adjoint 4D-Var & Observation Sensitivity

$$\Delta I_{\mathbf{a}} \approx \delta \mathbf{y}^{\mathrm{T}} \boxed{\left(\partial K / \partial \mathbf{y} \right)^{\mathrm{T}}} \left(\partial I / \partial \mathbf{z} \right)$$

Adjoint of
4D-Var

Observation System Experiments (OSEs)

Suppose that during a particular assimilation cycle the satellite altimeter goes offline.

How would this have impacted the analysis?

We could run 4D-Var again with SSH obs removed.

Or let $\delta y_i = -d_i$ **for all SSH obs.**

The change in the analysis is: $\delta \mathbf{z_a} \approx \left(\partial \mathcal{K} / \partial \mathbf{y} \right) \delta \mathbf{y}$

The change in $\Delta I_{\mathbf{a}}$ **is:** $\Delta I_{\mathbf{a}} \approx \delta \mathbf{y}^{\mathrm{T}} \left(\partial \mathcal{K} / \partial \mathbf{y} \right)^{\mathrm{T}} \left(\partial I / \partial \mathbf{z} \right)$

Observation System Experiments (OSEs)

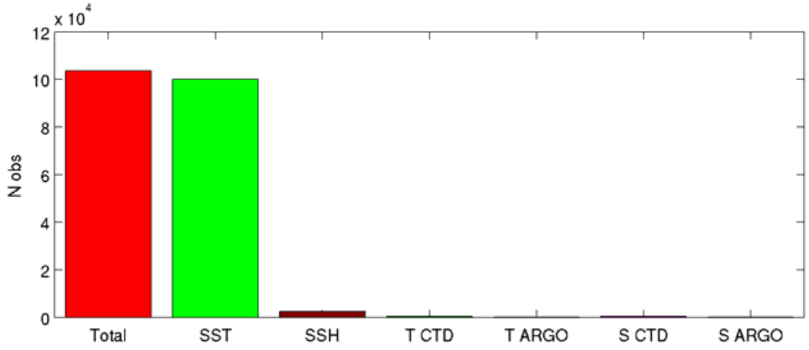
The cost of $(4D\text{-}Var)^T$ = cost of 4D-Var

But ONLY one run of $(4D\text{-}Var)^T$ is needed for ALL OSEs.

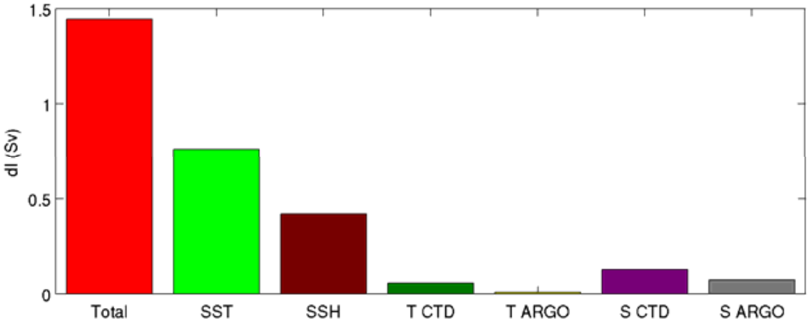
Example: 37N transport

(10km, CCS ROMS)

N_{obs}

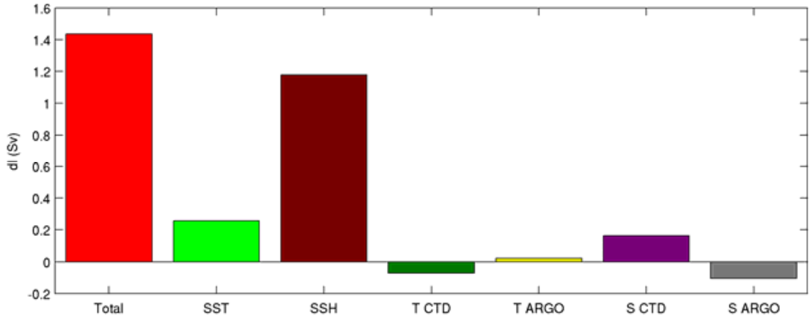


Obs Impact



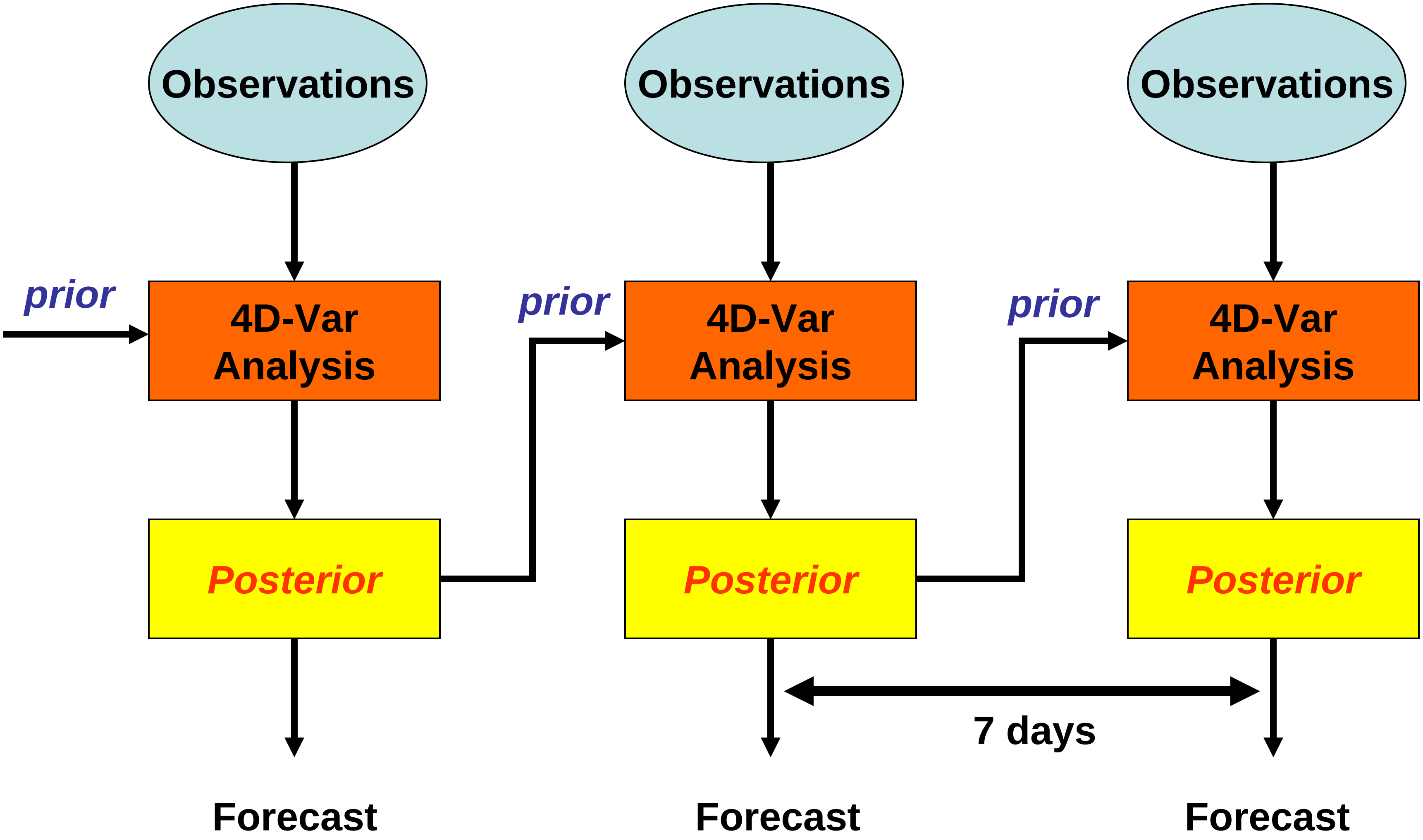
$$\Delta I = \mathbf{d}^{\mathrm{T}} \tilde{\mathbf{K}}^{\mathrm{T}} \sum_{i=1}^{N} \frac{1}{N} (\mathbf{M}_{\mathbf{b}})_i^{\mathrm{T}} \mathbf{h}_i$$

Obs Sens



$$\Delta I = \mathbf{d}^{\mathrm{T}} \left(\frac{\partial K}{\partial \mathbf{y}} \right)^{\mathrm{T}} \sum_{i=1}^{N} \frac{1}{N} (\mathbf{M}_{\mathbf{b}})_i^{\mathrm{T}} \mathbf{h}_i$$

Sequential 4D-Var with 30km CCS ROMS



Observations

Observations

Observations

prior

4D-Var Analysis

prior

4D-Var Analysis

prior

4D-Var Analysis

Posterior

Posterior

Posterior

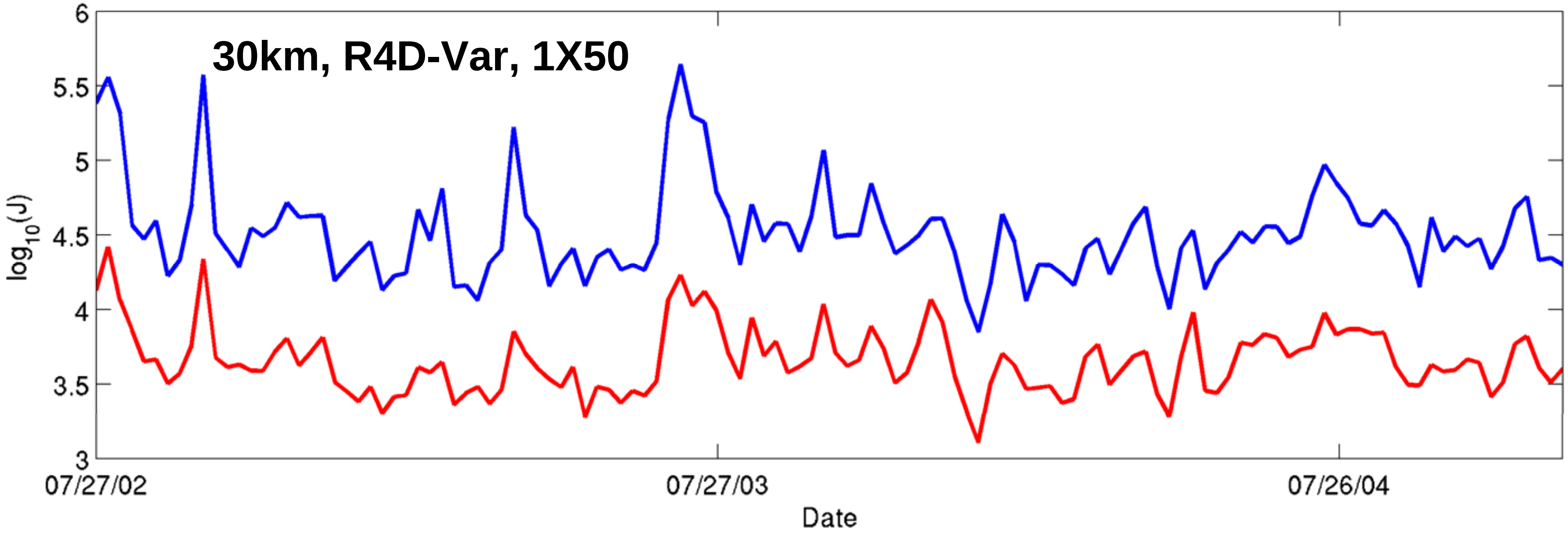
7 days

Forecast

Forecast

Forecast

Sequential 4D-Var CCS ROMS

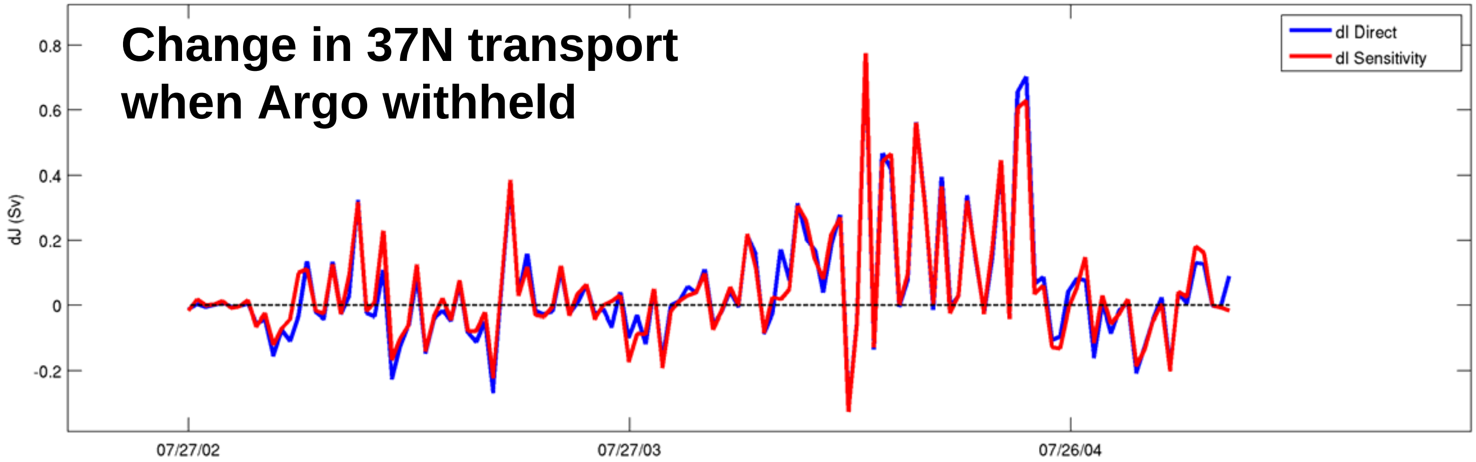


J initial

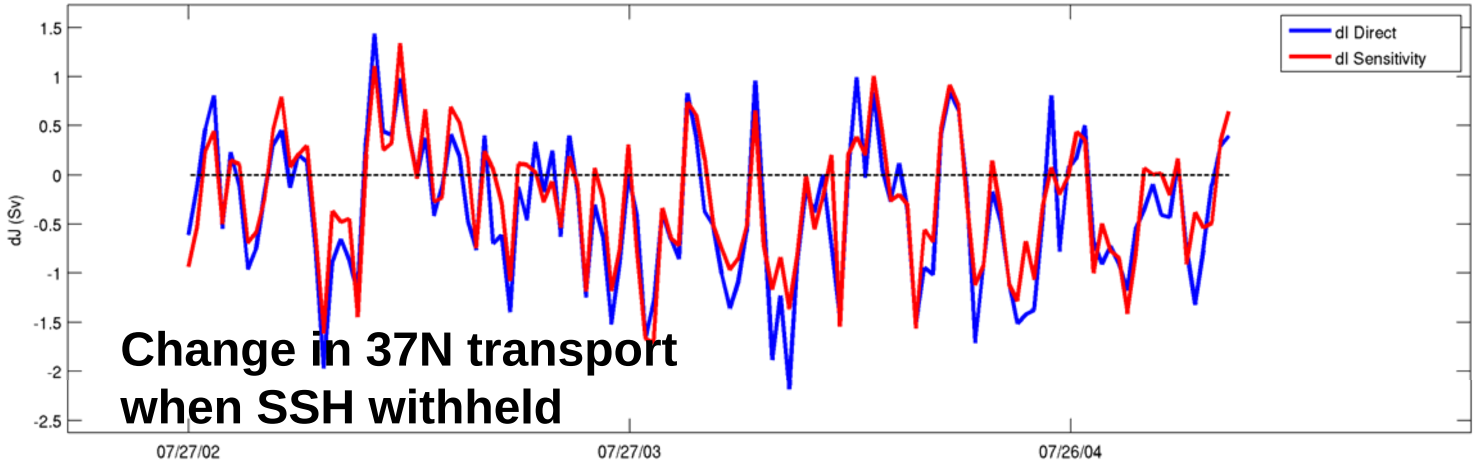
J final

Observing System Experiments (OSEs)

(30km, CCS ROMS)



Change in 37N transport when Argo withheld



Change in 37N transport when SSH withheld

Two Spaces: Obs Sensitivity

$\partial \mathcal{K}/\partial \mathbf{y}$ maps from observation (dual) space
to model (primal) space

$\left(\partial \mathcal{K}/\partial \mathbf{y}\right)^{\mathrm{T}}$ maps from model (primal) space
to observation (dual) space

Identifies the part of model space that controls 37N transport
and that is activated by the observations during 4D-Var

Observation Sensitivity: ROMS Implementation

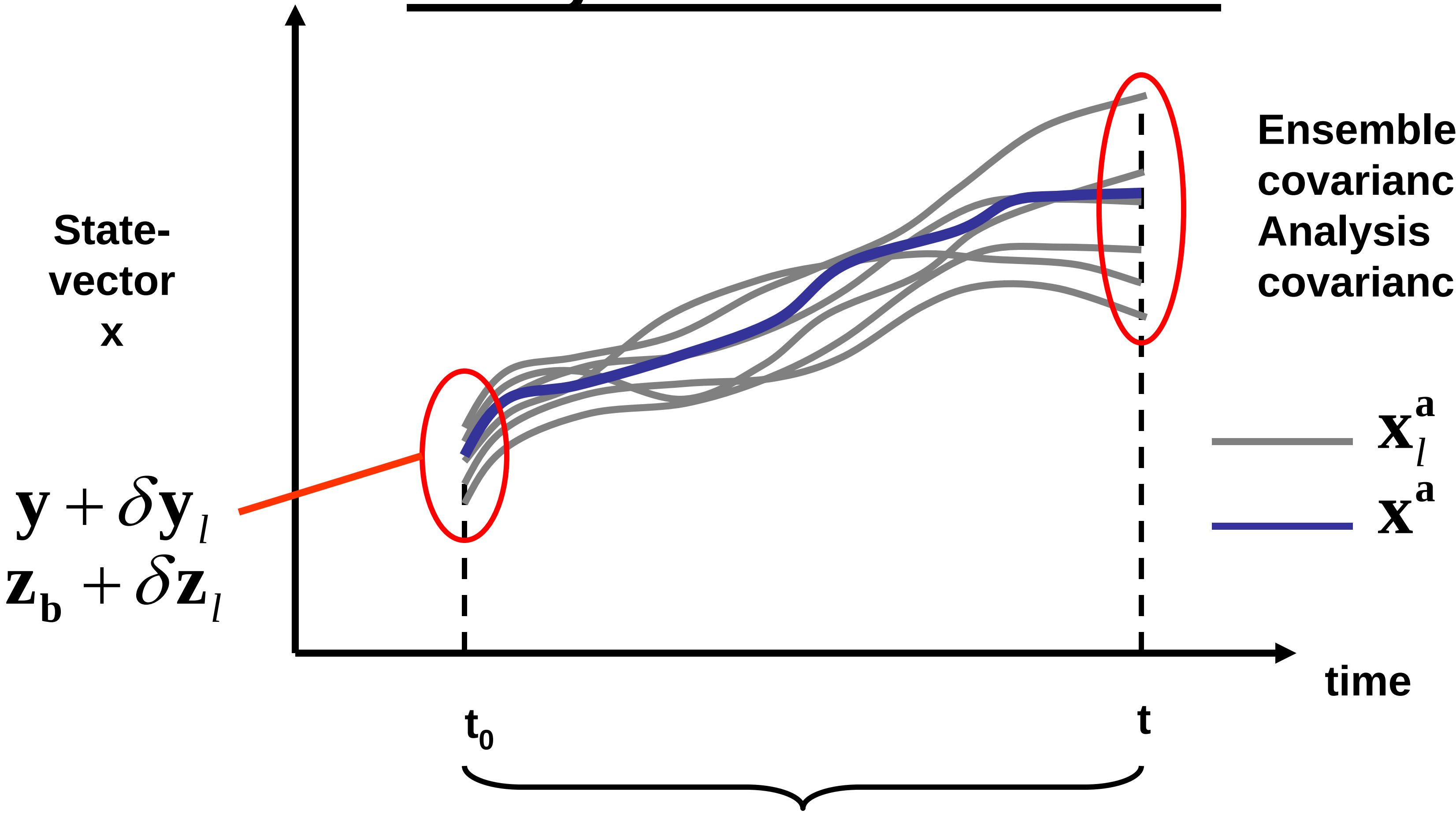
- **Dual (4D-PSAS & R4D-Var) forms only available:**

  - define W4DPSAS_SENSITIVITY
  (define RECOMPUTE_4DVAR)
  Drivers/obs_sen_w4dpsas.h

  - define W4DVAR_SENSITIVITY
  (define RECOMPUTE_4DVAR)
  Drivers/obs_sen_w4dvar.h

Error Covariance Estimates from $(\text{4D-Var})^T$

Analysis Error Revisited



Ensemble covariance= Analysis covariance

Ensemble of *M* 4D-Var analyses

Belo Pereira & Berre (2006):
(also Daget et al, 2010)

$$\mathbf{E}^\mathrm{a}=\frac{1}{M}\sum_{l=1}^{M}\left(\mathbf{x}_l^\mathrm{a}-\mathbf{x}^\mathrm{a}\right)\left(\mathbf{x}_l^\mathrm{a}-\mathbf{x}^\mathrm{a}\right)^\mathrm{T}$$

Analysis Error Revisited

An ensemble of 4D-Var analyses is very expensive!

But one run of (4D-Var)$^{\mathrm{T}}$ yields $\left(\partial\mathcal{K}/\partial\mathbf{d}\right)^{\mathrm{T}}$ **and:**

$$\delta\mathbf{z}_l^{\mathbf{a}} \approx \delta\mathbf{z}_l + \left(\partial\mathcal{K}/\partial\mathbf{d}\right)\delta\mathbf{d}_l; \qquad \delta\mathbf{d}_l \approx \delta\mathbf{y}_l + \mathbf{G}\delta\mathbf{z}_l$$

and: $\delta\mathbf{x}_l^{\mathbf{a}}(t) \approx \mathcal{M}(t,t_0)\delta\mathbf{z}_l^{\mathbf{a}}$

Therefore:

$$\mathbf{E}_{\mathbf{x}}^{\mathbf{a}}(t) = \left\langle \delta\mathbf{x}^{\mathbf{a}}(t)\left(\delta\mathbf{x}^{\mathbf{a}}(t)\right)^{\mathrm{T}} \right\rangle$$

$$= \mathcal{M}\left\{\left(\mathbf{I} - \left(\frac{\partial\mathcal{K}}{\partial\mathbf{d}}\right)\mathbf{G}\right)\mathbf{D}\left(\mathbf{I} - \left(\frac{\partial\mathcal{K}}{\partial\mathbf{d}}\right)\mathbf{G}\right)^{\mathrm{T}} + \left(\frac{\partial\mathcal{K}}{\partial\mathbf{d}}\right)\mathbf{R}\left(\frac{\partial\mathcal{K}}{\partial\mathbf{d}}\right)^{\mathrm{T}}\right\}\mathcal{M}^{\mathrm{T}}$$

Here, *M* is essentially infinite!

Analysis Error Revisited

For linear functions: $I(\mathbf{x}) = \sum_{k=1}^{N} \mathbf{h}_k^{\mathrm{T}} \mathbf{x}_k$

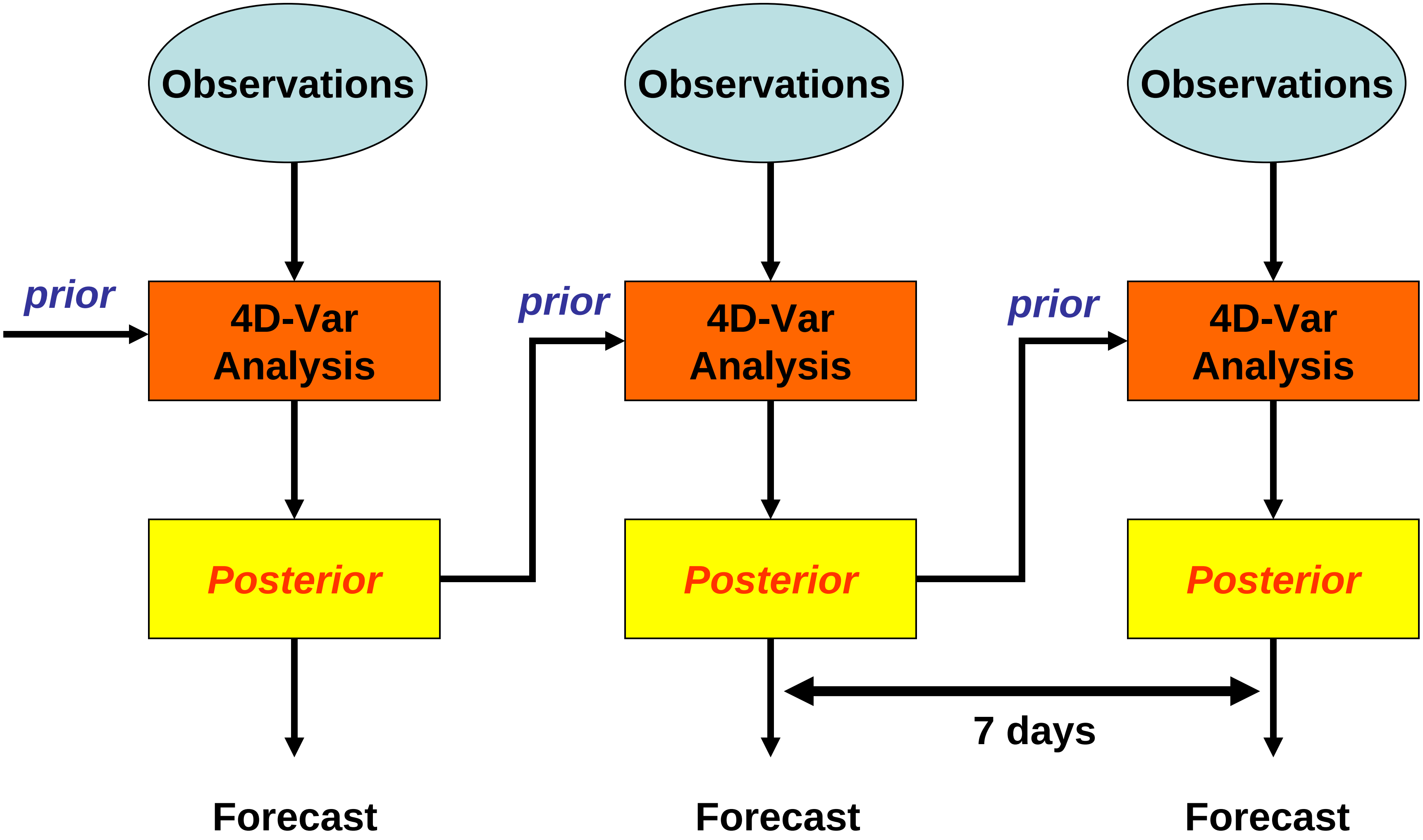
***Posterior*/analysis error variance:**

$$\left(\sigma_I^a\right)^2 = \left(\sum_{k=1}^{N} \mathbf{h}_k^{\mathrm{T}} \mathcal{M}_k\right) \mathbf{E}_{\mathbf{x}}^{\mathbf{a}}(t_0) \left(\sum_{j=1}^{N} \mathcal{M}_j^{\mathrm{T}} \mathbf{h}_j\right)$$

$$= \mathbf{g}^{\mathrm{T}} \mathbf{E}_{\mathbf{x}}^{\mathbf{a}}(t_0) \mathbf{g}$$

where: $\mathbf{g} = \sum_{j=1}^{N} \mathcal{M}_j^{\mathrm{T}} \mathbf{h}_j$ **(ADROMS forced by h)**

Sequential 4D-Var with 30km CCS ROMS



Observations

Observations

Observations

prior

4D-Var Analysis

prior

4D-Var Analysis

prior

4D-Var Analysis

Posterior

Posterior

Posterior

7 days

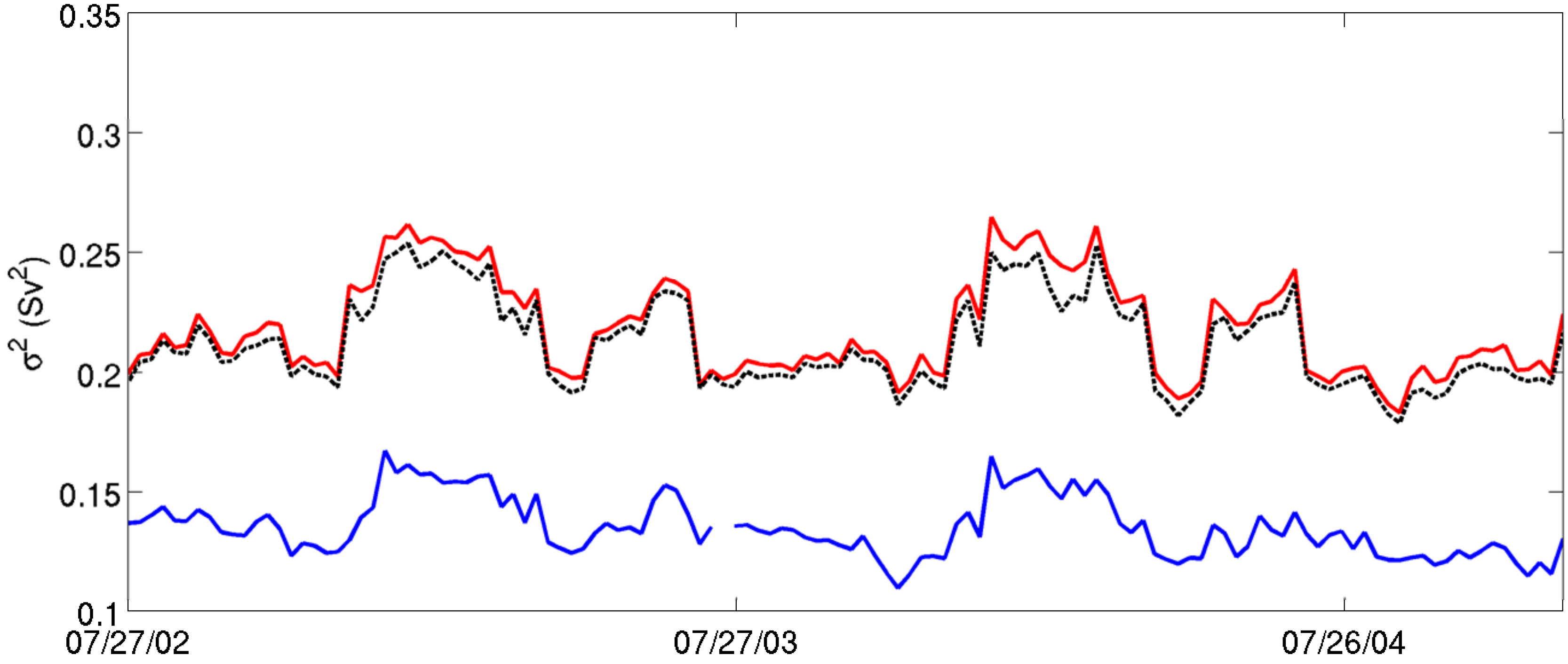
Forecast

Forecast

Forecast

Example: 37N Transport

30km, ROMS CCS, 7day average transport errors

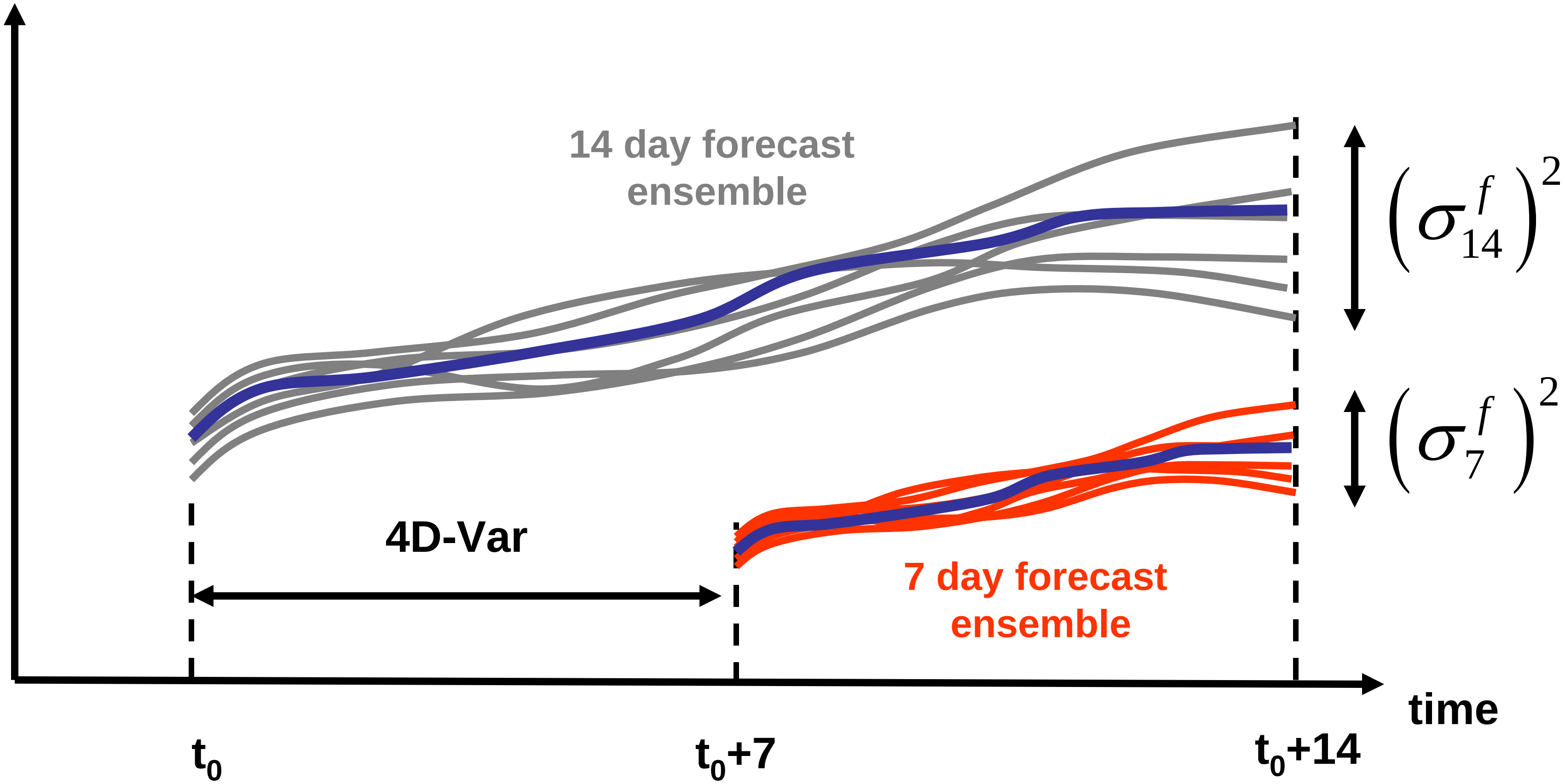


🟥 ***Prior* error**

🟦 ***Posterior* error using (4D-Var)^T**

…… ***Posterior* error using $\tilde{\mathrm{E}}^a$**

Predictability



Predictability due to assimilating observations during [t_0,t_0+7]:

$$\left(\sigma_{14}^{f}\right)^2 - \left(\sigma_{7}^{f}\right)^2$$

Example: 37N Transport Predictability

$\left(\sigma_{14}^{f}\right)$ = **spread of 14 day forecast ensemble of transport**

$\left(\sigma_{7}^{f}\right)$ = **spread of 7 day forecast ensemble of transport**

$$\left(\sigma_{14}^{f}\right)^{2}-\left(\sigma_{7}^{f}\right)^{2}=2\mathbf{g}^{\mathrm{T}}\mathbf{GD}\mathcal{M}_{\mathbf{b}}^{\mathrm{T}}\sum_{k}\left(\mathcal{M}_{14}\right)_{k}^{\mathrm{T}}\mathbf{h}_{k}-\mathbf{g}^{\mathrm{T}}\left(\mathbf{GDG}^{\mathrm{T}}+\mathbf{R}\right)\mathbf{g}$$

where: $\mathbf{g}=\left(\partial\mathcal{K}/\partial\mathbf{d}\right)^{\mathrm{T}}\mathcal{M}_{\mathbf{b}}^{\mathrm{T}}\sum_{k}\left(\mathcal{M}_{14}\right)_{k}^{\mathrm{T}}\mathbf{h}_{k}$

Seemingly complicated expressions, but really just TL and AD operators strung together in the right order!

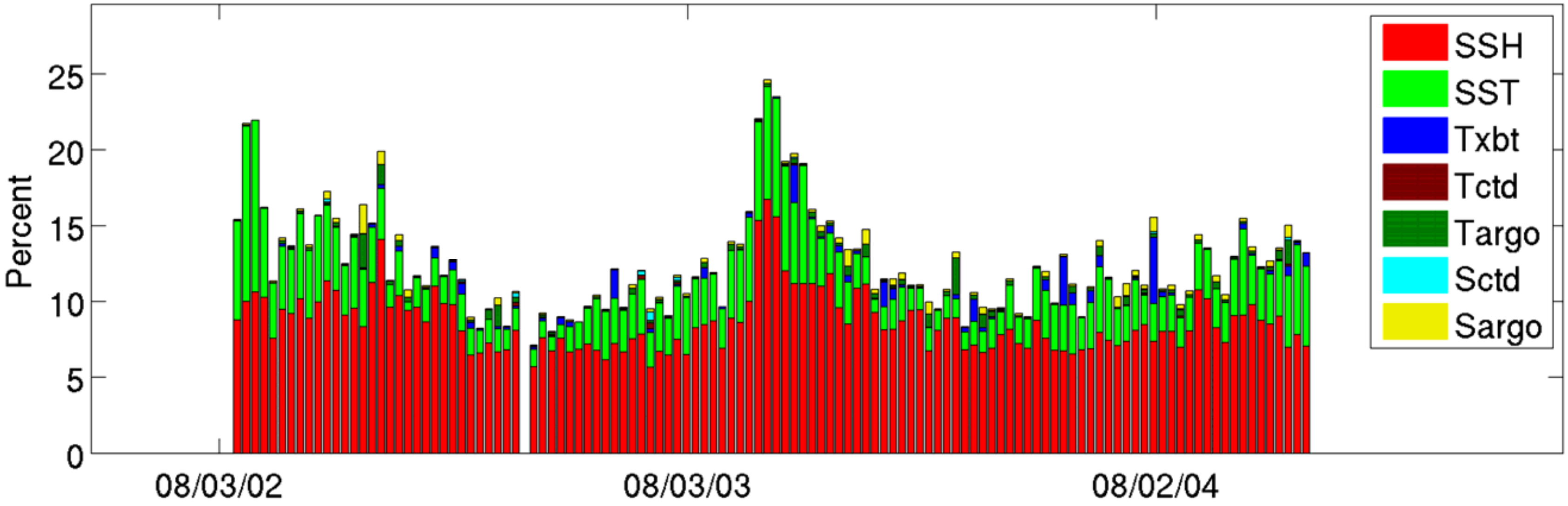
Example: 37N Transport Predictability

$$\left(\sigma_{14}^{f}\right)^{2}-\left(\sigma_{7}^{f}\right)^{2}=\boxed{2\mathbf{g}^{\mathrm{T}}\mathbf{G}\mathbf{D}\mathcal{M}_{\mathbf{b}}^{\mathrm{T}}\sum_{k}\left(\mathcal{M}_{14}\right)_{k}^{\mathrm{T}}\mathbf{h}_{k}}\boxed{-\mathbf{g}^{\mathrm{T}}\left(\mathbf{G}\mathbf{D}\mathbf{G}^{\mathrm{T}}+\mathbf{R}\right)\mathbf{g}}$$

🟥 **change in predictability due to the covariance between errors in the *priors* z_b and errors in the time evolving *prior* circulation $x_b(t)$ evaluated at the observation points.**

🟦 **change in predictability associated with the stabilized representer matrix - a combination of the covariance between errors in the time evolving *prior* circulation at the observation points, and the covariance between the observation errors (including errors of representativeness).**

Example: 37N Transport Predictability



$$r = 100\left\{\left(\sigma_{14}^{f}\right)^{2} - \left(\sigma_{7}^{f}\right)^{2}\right\}\Big/\left(\sigma_{14}^{f}\right)^{2}$$

$r > 0$ **implies 4D-Var increases predictability**

Issues, Things to do, & Coming Soon

- Observation sensitivity only available for dual 4D-Var.
- Observation impact and observation sensitivity calculations are currently restricted to a single outer-loop – multiple outer-loops coming soon.
- Increase the modularity of ROMS drivers so that arbitrary sequences of operators (linear and non-linear) can be formed.

Summary

- Observation impact is based on $\tilde{\mathbf{K}}^{\mathrm{T}}$ and yields the <u>actual</u> contribution of each obs to the circulation increments.
- Observation sensitivity is based on (4D-Var)^{T} and yields the change in circulation due to changes in obs (or array)
  - useful for efficient generation of OSEs.
- Both obs impact and obs sensitivity were applied in examples during analysis cycle, but can be applied during forecast cycle also (Moore et al, 2010c).
- (4D-Var)^{T} yields more reliable estimates of $\mathbf{E^a}$ and $\mathbf{E^f}$ and predictability.

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