Pressure Gradients under ice shelves
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Consider an ice shelf floating on an ocean with vertically varying density. Even under conditions of uniform in situ temperature and salinity, the in situ density will increase due to the pressure effect in the nonlinear equation of state. The ice shelf test case has initial conditions of constant and uniform temperature and salinity.

The ice shelf has a thickness $h_{\text{ice}} = \eta_{\text{ice}} - z_{\text{ice}}$ which is composed of a draft below sea level (z_{ice}, which is negative) and a freeboard (η_{ice}). The ice density is $\hat{\rho}_{\text{ice}}$.

Assume that the ice shelf has no shear strength and each ice column floats in the ocean. Then the hydrostatic balance requires that the weight of the ice shelf is equal to the weight of the water displaced or,

$$g\,\hat{\rho}_{\text{ice}}\,h_{\text{ice}} = g\int_{z_{\text{ice}}}^{0} \hat{\rho}(z)\,dz = g\int_{z_{\text{ice}}}^{0} (1000 + \rho(z))\,dz \tag{1}$$

where $\hat{\rho}$ is the total density and ρ is the density anomaly from 1000 kg m^{-3}, which is the convention used in ROMS.

The pressure in the water below the ice shelf ($z < z_{\text{ice}}$) is obtained from the hydrostatic balance as

$$P(x,y,z) = g\,\hat{\rho}_{\text{ice}}\,h_{\text{ice}} + g\int_{z}^{z_{\text{ice}}} (1000 + \rho(z))\,dz + g\,(1000 + \rho(z_{\text{ice}}))\,\eta \tag{2}$$

where the last term allows a pressure anomaly under the ice shelf in the form of a "sea surface elevation" times the local water density. This is the same formalism as is used in the ROMS model.

The ice density can be removed from consideration through the floating condition (1),

$$P(x,y,z) = g\int_{z_{\text{ice}}}^{0} (1000 + \rho(z))\,dz + g\int_{z}^{z_{\text{ice}}} (1000 + \rho(z))\,dz + g\,(1000 + \rho(z_{\text{ice}}))\,\eta, \tag{3}$$

where the first term is the pressure due to the ice shelf, the second term is the hydrostatic pressure under the ice and the third term is the pressure anomaly under the ice.

A uniform hydrostatic pressure based on a water density of 1000 kg m^{-3} is removed from ROMS to reduce pressure gradient errors. The expression for the residual pressure becomes

$$P(x,y,z) = g\int_{z_{\text{ice}}}^{0} \rho(z)\,dz + g\int_{z}^{z_{\text{ice}}} \rho(z)\,dz + g\,(1000 + \rho(z_{\text{ice}}))\,\eta, \tag{3}$$

nwhere the reference density is removed from the first and middle terms. The total weight of the ice must remain in the last expression.

Apply the condition that there should be no pressure gradient below the ice in the absence of a pressure perturbation ($\eta = 0$) or lateral variations in density then

$$\frac{\partial P}{\partial x} = 0 = -g\,\rho(z_{\mathrm{ice}})\,\frac{\partial z_{\mathrm{ice}}}{\partial x} + g\,\rho(z_{\mathrm{ice}})\,\frac{\partial z_{\mathrm{ice}}}{\partial x}.$$

So there are no horizontal pressure gradients in this formulation as long as the water density is a function only of depth.

The problem is now the integral through the ice shelf which is not part of the model. Simply imposing a constant ice density does not help as that would shift to problem to calculating the ice draft (η_{ice}) which would depend on the same distribution of density in the ocean which has been displaced.

One solution at the initial time is to calculate the integral along the base of the ice, where the density is known.

$$\int_{z_{\mathrm{ice}}}^{0} \rho(z)\,dz = \int_C \rho(z_{\mathrm{ice}})\nabla z_{\mathrm{ice}} \cdot d\vec{l},$$

along any curve C in the domain. If z_{ice} is continuously differentiable (which is true for reasonable ice shelves), then the integral depends only on the end points and not the path. If the path is defined by components $d\vec{l} = (r, s)$, then

$$\int_C \rho(z_{\mathrm{ice}})\nabla z_{\mathrm{ice}} \cdot d\vec{l}, = \int_C \rho(z_{\mathrm{ice}})(\frac{\partial z_{\mathrm{ice}}}{\partial r}\,dr + \frac{\partial z_{\mathrm{ice}}}{\partial s}\,ds)$$

If there is a region which is not ice covered, then the integral should start in the ice free region and integrate along (ξ, η) index lines until the domain is covered. For example, integrate from north to south along η lines if the north is ice free. Alternatively, integrate zonally along the northern boundary from the ice free region into the ice covered region, then southward along η lines.

If the whole domain is ice covered, then the thinnest ice constitutes a constant pressure which can be removed in the same way that a constant hydrostatic pressure is removed. Or the pressure can be calculated with the assumption that the ocean density is uniform ($= \hat{\rho}_{\mathrm{ice}}$) from the depth of the thinnest ice to the ocean surface.

A new 2D array (`IcePress`) is defined in mod_grid.F to hold the integrated (time invarient) pressure at the base of the ice shelf. Everywhere in mod_grid.F with `zice`, I did the same thing with `IcePress`. It is calculated as

$$\texttt{IcePress}(x, y) = \int_C \rho(z_{\mathrm{ice}})(\frac{\partial z_{\mathrm{ice}}}{\partial r}\,dr + \frac{\partial z_{\mathrm{ice}}}{\partial s}\,ds)$$

along appropriate paths in the domain. This term then appears in the pressure calculation as ... + `GRho*IcePress(i,j)` ... or in prsgrd32.F

```
P(i,j,N(ng))
      =GRho0*(z_w(i,j,N(ng))-zice(i,j))
&             +GRho*rho(i,j,N(ng))*(z_w(i,j,N(ng))-zice(i,j))
&             +GRho*IcePress(i,j))+
&             ...
```

Within initial.F, the pressure is calculated as

```
!  this assumes that the zice is the same along the northern boundary
      do i=LBi,UBi
            GRID(ng)%IcePress(i,UBj)=0.0_r8
            do j=UBj-1,LBj,-1
                  GRID(ng)%IcePress(i,j)=GRID(ng)%IcePress(i,j+1)+0.5_r8
     &            *(OCEAN(ng)%rho(i,j+1,N(ng))+OCEAN(ng)%rho(i,j,N(ng)))
     &            *(GRID(ng)%zice(i,j+1)-GRID(ng)%zice(i,j))
            enddo
      enddo
```

This calculation is done after rho_eos is called so that the water density under the ice is known. I had to define local integers i, j.