Tutorial 9: Computing Normalization Coefficients for Covariance Models
**Prior Error Covariance Modeling**

Recall: \( B_x = K_b \Sigma C \Sigma^T K_b^T \)

C is a correlation matrix for the unbalanced increments, and is modeled as the solution of a diffusion equation:

\[
\frac{\partial x}{\partial t} - \kappa \nabla^2 x = 0 \quad \rightarrow \quad C'x
\]

But, \( C' \) is arbitrary at this stage, and must to be normalized to ensure that the range is ±1 as required for a correlation function. Therefore we define:

\[
C = \Lambda C' \Lambda^T
\]

where \( \Lambda \) is a diagonal matrix with elements \( (c'_{ii})^{-1/2} \)

Finally: \( B_x = K_b \Sigma \Lambda C' \Lambda^T \Sigma^T K_b^T \)
Computing $\Lambda$
(define NORMALIZATION)

Following Weaver & Courtier (2001) we employ two methods for computing the elements of $\Lambda$:

(i) Exact method (Nmethod=0):

$$C'e_i \rightarrow i^{th} \text{ column of } C'; \text{ save } c_{ii}$$

where $e_i^T = (0, 0, \ldots, 0, 1, 0, \ldots, 0)$

$\uparrow$

$i^{th}$ element

Requires $N_{grid}$ runs of diffusion operators: impractical for v. large grids.
(ii) Randomization method (N\text{method}=1):

Estimate the diagonal elements $c'_{ii}$ of $C'$ from:

$$C' \approx \frac{1}{M} \sum_{i=1}^{M} \xi C' \xi^T = \tilde{C}$$

where $\xi$ is a random vector: $\xi \rightarrow N(0, I)$

$M$ is the sample size. As $M \rightarrow \infty$, $\tilde{C} \rightarrow C'$

Uncertainty in elements of $\Lambda^{-1}$: $(2M)^{-1/2}$

Practical requirement: $M << N_{\text{grid}}$

(Fisher and Courtier, 1995)
Computing $\Lambda$

Practicalities:

- choose $M$ and run normalization driver
- choose another seed and run again
- compare the estimates – are they similar
- compute means of the two $M$ samples
- repeat as necessary until mean does not change significantly
Error Covariance Normalization

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Introduction [edit]

In this tutorial you will compute the 4D-Var error covariance (D) normalization factors for the California Current System application WC13.

The error covariance matrix, $D = diag(B, B_{\theta}, B_{\theta}, Q)$, is very large and not well known. $B$ and $Q$ are modeled as the solution of a diffusion equation following Weaver and Courtier (2001) measure. Each covariance matrix is factorized as $B = X \Sigma X^T$, where $\Sigma$ is a univariate correlation matrix, $X$ is a diagonal matrix of error standard deviations, and $K$ is a multivariate balance operator. The normalization coefficients are needed to ensure that the diagonal elements of the associated correlation matrix $C$ are equal to unity.

There are two methods to compute the error covariance normalization coefficients: exact and randomization (an approximation).

The exact method is very expensive on large grids. The normalization coefficients are computed by perturbing each model grid cell with a delta function scaled by the area (2D state variables) or volume (3D state variables), and then by convolving with the squared-root adjoint and tangent linear diffusion operators.

In the cheaper approximate method, the normalization coefficients are computed using the randomization approach of Fisher and Courtier (1995). The coefficients are initialized with random numbers having a uniform distribution (drawn from a normal distribution with zero mean and unit variance). Then, they are scaled by the inverse squared-root of the cell area (2D state variable) or volume (3D state variable) and convolved with the squared-root adjoint and tangent diffusion operators over a specified number of iterations, $N_{\text{random}}$.

Since the grid for WC13 is relatively small, the error covariance normalization coefficients are computed using the exact method. They need to be computed only once for a particular application provided that the grid, land/sea masking (if any), and decorrelation scales remain the same.

Model Set-up [edit]

The WC13 model domain is shown in Fig. 1, and the area covered by the northern, western, and southern edges of the model domain.
WC13 C-preprocessing Options
(Basic Configuration)

Momentum Equations Options:

`#define UV_ADV` including advection terms
`#define UV_COR` including Coriolis term
`#define UV_U3ADVECTION` 3rd-order Upstream Horizontal advection
`#define UV_C4ADVECTION` 4th-order Centered Vertical advection
`#define DJ_GRADPS` splines density Jacobian PGF
`#define UV_QDRAG` quadratic bottom friction
`#define MIX_S_UV` mixing along s-levels
`#define SPLINES_WVISC` parabolic Splines for Vertical Viscosity

Tracers Equations Options:

`#define TS_U3ADVECTION` 3rd-order Upstream Horizontal advection
`#define TS_C4ADVECTION` 4th-order Centered advection
`#define TS_DIF2` harmonic horizontal mixing
`#define MIX_GEO_TS` mixing along geo-potentials
`#define SALINITY` including salinity
`#define NONLIN_EOS` nonlinear equation of state
`#define SPLINES_VDIFF` parabolic splines for vertical Diffusion

Surface Forcing Options:

`#define BULK_FLUXES` surface bulk fluxes parameterization
`#define DIURNAL_SRFLUX` modulate shortwave by the local diurnal cycle
`#define EMINUSP` compute Salt Flux using E-P
`#define LONGWAVE_OUT` compute outgoing longwave radiation
`#define SOLAR_SOURCE` solar radiation source term

Vertical Turbulent Mixing Parameterization Options:

`#define GLS_MIXING` Generic Length Scale Mixing (K-omega)
`#ifdef GLS_MIXING` # define N2S2_HORAVG smoothing of buoyancy/shear
`#define KANTHA_CLAYSON` stability function
`#define RI_SPLINES` parabolic splines for Ri Number

Model Configuration Options:

`#define SOLVE3D` solve 3D primitive equations
`#define CURVGRID` curvilinear grid
`#define MASKING` land/sea masking
`#define SPHERICAL` spherical grid
`#define PROFILE` time profiling
`#define ANA_SPONGE` analytical viscosity/diffusion sponge
WC13 C-preprocessing Options
(Error Covariance Configuration)

Algorithm:
#define NORMALIZATION primal form of incremental strong constraint 4D-Var
#define ANA_INITIAL analytical initial conditions (zero fields)

Control Vector:
#define ADJUST_BOUNDARY open boundary conditions increments
#define ADJUST_STFLUX surface tracer flux increments
#define ADJUST_WSTRESS surface wind stress increments

Error Covariance Modeling:
#define CORRELATION model error covariance correlation with diffusion operators
#define FULL_GRID consider both interior and boundary points
#define VCONVOLUTION Vertical correlation modeling
#define IMPLICIT_VCON Implicit vertical diffusion operator
#undef BALANCE_OPERATOR Multivariate balance constraint
#define ZETA_ELLIPTIC SSH elliptic equation method

Prior:
#define FORWARD_READ read basic state linearization in TLM and ADM files
#define FORWARD_WRITE writing basic state by the NLM
#define FORWARD_MIXING processing basic state vertical mixing coefficients
#define NL_BULK_FLUXES surface kinematic fluxes from nonlinear model

I/O:
#define OUT_DOUBLE double precision data in output NLM, TLM, and ADM
Options for the California Current System, 1/3 degree resolution.

Application flag: WC13
Input script: roms_wc13.in

Available Drivers options: choose only one and activate it in the build.sh script (MY_CPP_FLAGS definition)

AD_SENSITIVITY
AFT_EIGENMODES
ARRAY_MODES
CLIPPING
CORRELATION
GRADIENT_CHECK
FORCING_SV
FT_EIGENMODES
I4DVAR
NLM_DRIVER
OPT_PERTURBATION
PICARD_TEST
RBL4DVAR

Adjoint Sensitivity Driver
Adjoint Finite Time Eigenmodes
Stabilized representer matrix array modes
Stabilized representer matrix clipped analysis
Background-error Correlation Check
TLM/ADM Gradient Check
Forcing Singular Vectors
Finite Time Eigenmodes
Incremental, strong constraint I4D-Var
Nonlinear Basic State trajectory
Optimal perturbations
Picard Iterations Test
Strong/Weak constraint RBL4D-Var
Four different error covariance normalization coefficients NetCDF files are required in ROMS 4D-Var algorithms to ensure that the diagonal elements of the associated correlation matrix \( (C) \) are equal to unity:

- Model error normalization file, if weak constraint
- Initial conditions normalization file
- Open boundary conditions normalization file, if \( \text{ADJUST\_BOUNDARY} \)
- Surface forcing normalization file, if \( \text{ADJUST\_WSTRESS} \) and/or \( \text{ADJUST\_STFLUX} \)

These normalization NetCDF files are specified in 4D-Var input script as:

\[
\begin{align*}
\text{NRMnameM} & = \text{../Data/wc13_nrm_m.nc} \\
\text{NRMnameI} & = \text{../Data/wc13_nrm_i.nc} \\
\text{NRMnameB} & = \text{../Data/wc13_nrm_b.nc} \\
\text{NRMnameF} & = \text{../Data/wc13_nrm_f.nc}
\end{align*}
\]
Model Error and Initial Conditions Metadata

Variables:

```plaintext
double zeta(ocean_time, eta_rho, xi_rho);
    zeta:long_name = "free-surface, initial conditions error covariance normalization";
    zeta:units = "meter";
    zeta:time = "ocean_time";
    zeta:coordinates = "lon_rho lat_rho ocean_time"

doubleubar(ocean_time, eta_u, xi_u);
    ubar:long_name = "vertically integrated u-momentum component, initial conditions error covariance normalization";
    ubar:units = "meter";
    ubar:time = "ocean_time";
    ubar:coordinates = "lon_u lat_u ocean_time"

double vbar(ocean_time, eta_v, xi_v);
    vbar:long_name = "vertically integrated v-momentum component, initial conditions error covariance normalization";
    vbar:units = "meter";
    vbar:time = "ocean_time";
    vbar:coordinates = "lon_v lat_v ocean_time"

double u(ocean_time, s_rho, eta_u, xi_u);
    u:long_name = "u-momentum component, initial conditions error covariance normalization";
    u:units = "meter";
    u:time = "ocean_time";
    u:coordinates = "lon_u lat_u s_rho ocean_time"

double v(ocean_time, s_rho, eta_v, xi_v);
    v:long_name = "v-momentum component, initial conditions error covariance normalization";
    v:units = "meter";
    v:time = "ocean_time";
    v:coordinates = "lon_v lat_v s_rho ocean_time"

double temp(ocean_time, s_rho, eta_rho, xi_rho);
    temp:long_name = "potential temperature, initial conditions error covariance normalization";
    temp:units = "meter";
    temp:time = "ocean_time";
    temp:coordinates = "lon_rho lat_rho s_rho ocean_time"

double salt(ocean_time, s_rho, eta_rho, xi_rho);
    salt:long_name = "salinity, initial conditions error covariance normalization";
    salt:units = "meter";
    salt:time = "ocean_time";
    salt:coordinates = "lon_rho lat_rho s_rho ocean_time"
```
dimensions:

\[
\begin{align*}
\text{xi}_\text{rho} &= 56 \\
\text{eta}_\text{rho} &= 55 \\
\ldots
\end{align*}
\]

boundary = 4

variables:

\[
\begin{align*}
\text{double } \text{zeta}_\text{obc}(\text{ocean\_time}, \text{boundary}, \text{IorJ}) ; \\
\text{zeta}_\text{obc}:\text{long\_name} &= \text{"free-surface, open boundaries conditions error covariance normalization"} \\
\text{zeta}_\text{obc}:\text{units} &= \text{"meter"} \\
\text{zeta}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\text{double } \text{ubar}_\text{obc}(\text{ocean\_time}, \text{boundary}, \text{IorJ}) ; \\
\text{ubar}_\text{obc}:\text{long\_name} &= \text{"vertically integrated u-momentum component, open boundaries conditions error covariance normalization"} \\
\text{ubar}_\text{obc}:\text{units} &= \text{"meter second\^{-1}"} \\
\text{ubar}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\text{double } \text{vbar}_\text{obc}(\text{ocean\_time}, \text{boundary}, \text{IorJ}) ; \\
\text{vbar}_\text{obc}:\text{long\_name} &= \text{"vertically integrated v-momentum component, open boundaries conditions error covariance normalization"} \\
\text{vbar}_\text{obc}:\text{units} &= \text{"meter second\^{-1}"} \\
\text{vbar}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\text{double } \text{u}_\text{obc}(\text{ocean\_time}, s\_\text{rho}, \text{boundary}, \text{IorJ}) ; \\
\text{u}_\text{obc}:\text{long\_name} &= \text{"u-momentum component, open boundaries conditions error covariance normalization"} \\
\text{u}_\text{obc}:\text{units} &= \text{"meter second\^{-1}"} \\
\text{u}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\text{double } \text{v}_\text{obc}(\text{ocean\_time}, s\_\text{rho}, \text{boundary}, \text{IorJ}) ; \\
\text{v}_\text{obc}:\text{long\_name} &= \text{"v-momentum component, open boundaries conditions error covariance normalization"} \\
\text{v}_\text{obc}:\text{units} &= \text{"meter second\^{-1}"} \\
\text{v}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\text{double } \text{temp}_\text{obc}(\text{ocean\_time}, s\_\text{rho}, \text{boundary}, \text{IorJ}) ; \\
\text{temp}_\text{obc}:\text{long\_name} &= \text{"potential temperature, open boundaries conditions error covariance normalization"} \\
\text{temp}_\text{obc}:\text{units} &= \text{"Celsius"} \\
\text{temp}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\text{double } \text{salt}_\text{obc}(\text{ocean\_time}, s\_\text{rho}, \text{boundary}, \text{IorJ}) ; \\
\text{salt}_\text{obc}:\text{long\_name} &= \text{"salinity, open boundaries conditions error covariance normalization"} \\
\text{salt}_\text{obc}:\text{time} &= \text{"ocean\_time"} \\
\end{align*}
\]

// global attributes:

\[
\text{boundary\_index} = \text{"West=1, South=2, East=3, North=4"}
\]
Surface Forcing Metadata

dimensions:
  xi_rho = 56;
  xi_u = 55;
  xi_v = 56;
  eta_rho = 55;
  eta_u = 55;
  eta_v = 54;
  s_rho = 30;
  ocean_time = UNLIMITED ; // (1 currently)

variables:
  
  double sustr(ocean_time, eta_u, xi_u);
  sustr:long_name = "surface u-momentum stress, error covariance normalization";
  sustr:units = "newton meter-2";
  sustr:time = "ocean_time";
  sustr:coordinates = "lon_u lat_u ocean_time";

  double svstr(ocean_time, eta_v, xi_v);
  svstr:long_name = "surface v-momentum stress, error covariance normalization";
  svstr:units = "newton meter-2";
  svstr:time = "ocean_time";
  svstr:coordinates = "lon_v lat_v ocean_time";

  double shflux(ocean_time, eta_rho, xi_rho);
  shflux:long_name = "surface net heat flux, error covariance normalization";
  shflux:units = "watt meter-2";
  shflux:negative = "upward flux, cooling";
  shflux:positive = "downward flux, heating";
  shflux:time = "ocean_time";
  shflux:coordinates = "lon_rho lat_rho ocean_time";

  double ssflux(ocean_time, eta_rho, xi_rho);
  ssflux:long_name = "surface net salt flux (E-P)*SALT, error covariance normalization";
  ssflux:units = "meter second-1";
  ssflux:time = "ocean_time";
  ssflux:coordinates = "lon_rho lat_rho ocean_time";

// global attributes:

  :type = "ROMS 4D-Var surface forcing error covariance normalization";
  :title = "California Current System, 1/3 degree resolution (WC13)";
  :Conventions = "CF-1.4";
  :grd_file = "test/WC13/Data/wc13_grd.nc";
ROMS/TOMS Standard Input parameters.

vn $Id: roms_wc13.in 971 2019-07-09 03:02:57Z arango $

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See License_ROMS.txt

Input parameters can be entered in ANY order, provided that the parameter KEYWORD (usually, upper case) is typed correctly followed by "=" or "==" symbols. Any comment lines are allowed and must begin with an exclamation mark (!) in column one. Comments may appear to the right of a parameter specification to improve documentation. Comments are ignored during reading. Blank lines are also allowed and ignored. Continuation lines in a parameter specification are allowed if preceded by a backslash (\). In some instances, more than one value is required for a parameter. If fewer values are provided, the last value is assigned for the entire parameter array. The multiplication symbol (*), without blank spaces in between, is allowed for a parameter specification. For example, in two grids nested application:

AKT_BAK == 2*1.0d-6 2*5.0d-6 ! m2/s

indicates that the first two entries of array AKT_BAK, in fortran column-major order, will have the same value of "1.0d-6" for grid 1, whereas the next two entries will have the same value of "5.0d-6" for grid 2.

In multiple levels of nesting or multiple connected domains step-ups...
Important Parameters

\begin{itemize}
\item Nmethod == 0  
\hspace{1em} normalization method: \(0=\text{Exact (expensive)}\) or \(1=\text{Approximated (randomization)}\)  
\item Nrandom == 5000  
\hspace{1em} randomization iterations  
\item LdefNRM == F F F F  
\hspace{1em} Create a new normalization files (model, IC, OBC, surface forcing)  
\item LwrtNRM == F F F F  
\hspace{1em} Compute and write normalization (model, IC, OBC, surface forcing)  
\item CnormM(isFsur) = T  
\hspace{1em} model error covariance, 2D variable at RHO-points  
\item CnormM(isUbar) = T  
\hspace{1em} model error covariance, 2D variable at U-points  
\item CnormM(isVbar) = T  
\hspace{1em} model error covariance, 2D variable at V-points  
\item CnormM(isUvel) = T  
\hspace{1em} model error covariance, 3D variable at U-points  
\item CnormM(isVvel) = T  
\hspace{1em} model error covariance, 3D variable at V-points  
\item CnormM(isTvar) = T T  
\hspace{1em} model error covariance, NT tracers  
\item CnormI(isFsur) = T  
\hspace{1em} IC error covariance, 2D variable at RHO-points  
\item CnormI(isUbar) = T  
\hspace{1em} IC error covariance, 2D variable at U-points  
\item CnormI(isVbar) = T  
\hspace{1em} IC error covariance, 2D variable at V-points  
\item CnormI(isUvel) = T  
\hspace{1em} IC error covariance, 3D variable at U-points  
\item CnormI(isVvel) = T  
\hspace{1em} IC error covariance, 3D variable at V-points  
\item CnormI(isTvar) = T T  
\hspace{1em} IC error covariance, NT tracers  
\item CnormB(isFsur) = T  
\hspace{1em} OBC error covariance, 2D variable at RHO-points  
\item CnormB(isUbar) = T  
\hspace{1em} OBC error covariance, 2D variable at U-points  
\item CnormB(isVbar) = T  
\hspace{1em} OBC error covariance, 2D variable at V-points  
\item CnormB(isUvel) = T  
\hspace{1em} OBC error covariance, 3D variable at U-points  
\item CnormB(isVvel) = T  
\hspace{1em} OBC error covariance, 3D variable at V-points  
\item CnormB(isTvar) = T T  
\hspace{1em} OBC error covariance, NT tracers  
\item CnormF(isUstr) = T  
\hspace{1em} surface forcing error covariance, U-momentum stress  
\item CnormF(isVstr) = T  
\hspace{1em} surface forcing error covariance, V-momentum stress  
\item CnormF(isTsur) = T T  
\hspace{1em} Surface forcing error covariance, NT tracers fluxes  
\item NRNameM == wc13_nrm_m.nc  
\hspace{1em} model error (weak constraint)  
\item NRNameI == wc13_nrm_i.nc  
\hspace{1em} initial conditions  
\item NRNameB == wc13_nrm_b.nc  
\hspace{1em} open boundary conditions  
\item NRNameF == wc13_nrm_f.nc  
\hspace{1em} surface forcing (wind stress and net heat flux)  
\end{itemize}
Horizontal and vertical stability and accuracy factors (< 1):

<table>
<thead>
<tr>
<th>Model</th>
<th>IC</th>
<th>Model</th>
<th>OBC</th>
<th>Sur</th>
<th>For</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hgamma = 0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>! horizontal operator</td>
<td></td>
</tr>
<tr>
<td>Vgamma = 0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>! vertical operator</td>
<td></td>
</tr>
</tbody>
</table>

Model error correlations (m):
- $H_{\text{decayM}(\text{isFsur})} = 50.0d+3$ ! free-surface (16 convolutions)
- $H_{\text{decayM}(\text{isUbar})} = 50.0d+3$ ! 2D U-momentum (16 convolutions)
- $H_{\text{decayM}(\text{isVbar})} = 50.0d+3$ ! 2D V-momentum (16 convolutions)
- $H_{\text{decayM}(\text{isUvel})} = 50.0d+3$ ! 3D U-momentum (16 convolutions)
- $H_{\text{decayM}(\text{isVvel})} = 50.0d+3$ ! 3D V-momentum (16 convolutions)
- $H_{\text{decayM}(\text{isTvar})} = 50.0d+3$ ! 1:NT tracers (16 convolutions)
- $V_{\text{decayM}(\text{isUvel})} = 30.0d0$ ! 3D U-momentum (8 convolutions)
- $V_{\text{decayM}(\text{isVvel})} = 30.0d0$ ! 3D V-momentum (8 convolutions)
- $V_{\text{decayM}(\text{isTvar})} = 30.0d0$ ! 1:NT tracers (8 convolutions)

Initial conditions error correlations (m):
- $H_{\text{decayI}(\text{isFsur})} = 50.0d+3$ ! free-surface (16 convolutions)
- $H_{\text{decayI}(\text{isUbar})} = 50.0d+3$ ! 2D U-momentum (16 convolutions)
- $H_{\text{decayI}(\text{isVbar})} = 50.0d+3$ ! 2D V-momentum (16 convolutions)
- $H_{\text{decayI}(\text{isUvel})} = 50.0d+3$ ! 3D U-momentum (16 convolutions)
- $H_{\text{decayI}(\text{isVvel})} = 50.0d+3$ ! 3D V-momentum (16 convolutions)
- $H_{\text{decayI}(\text{isTvar})} = 50.0d+3$ ! 1:NT tracers (16 convolutions)
- $V_{\text{decayI}(\text{isUvel})} = 30.0d0$ ! 3D U-momentum (8 convolutions)
- $V_{\text{decayI}(\text{isVvel})} = 30.0d0$ ! 3D V-momentum (8 convolutions)
- $V_{\text{decayI}(\text{isTvar})} = 30.0d0$ ! 1:NT tracers (8 convolutions)

Surface forcing error correlations (m):
- $H_{\text{decayF}(\text{isUstr})} = 100.0d+3$ ! surface U-momentum stress (66 convolutions)
- $H_{\text{decayF}(\text{isVstr})} = 100.0d+3$ ! surface V-momentum stress (66 convolutions)
- $H_{\text{decayF}(\text{isTsur})} = 100.0d+3$ ! 1:NT surface tracer flux (66 convolutions)
The exact method is very expensive on large grids.

The normalization coefficients are computed by perturbing each model grid cell with a delta function scaled by the area (2D state variables) or volume (3D state variables), and then convolving with the squared-root adjoint and tangent linear diffusion operators. The diffusion operator is self-adjointed.

The randomization method is cheaper (Fisher and Courtier, 1985).

The normalization coefficients are initialized with random numbers having a uniform distribution (drawn from a normal distribution with zero mean and unit variance). Then, they are scaled by the inverse squared-root of the cell area (2D state variables) or volume (3D state variables) and convolved with the squared-root adjoint and tangent linear diffusion operator over a specified number of iterations, \( N_{\text{random}} \).

The normalization coefficients need to be computed only once for a particular application provided that the grid, land/sea masking (if any), and decorrelation scales remain the same.
4D-Var Parameters: Decorrelation Scales

Horizontal and vertical stability and accuracy factors (< 1):

<table>
<thead>
<tr>
<th></th>
<th>IC</th>
<th>Model</th>
<th>OBC</th>
<th>Sur For</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{\gamma}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$V_{\gamma}$</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Model Error correlations (m):

- $H_{\text{decayM}(isFsur)}$ = $50.0d+3$ ! free-surface (16 convolutions)
- $H_{\text{decayM}(isUbar)}$ = $50.0d+3$ ! 2D U-momentum (16 convolutions)
- $H_{\text{decayM}(isVbar)}$ = $50.0d+3$ ! 2D V-momentum (16 convolutions)
- $H_{\text{decayM}(isUvel)}$ = $50.0d+3$ ! 3D U-momentum (16 convolutions)
- $H_{\text{decayM}(isVvel)}$ = $50.0d+3$ ! 3D V-momentum (16 convolutions)
- $H_{\text{decayM}(isTvar)}$ = $50.0d+3$ ! 1:NT tracers (16 convolutions)

- $V_{\text{decayM}(isUvel)}$ = $30.0d0$ ! 3D U-momentum (8 convolutions)
- $V_{\text{decayM}(isVvel)}$ = $30.0d0$ ! 3D V-momentum (8 convolutions)
- $V_{\text{decayM}(isTvar)}$ = $30.0d0$ ! 1:NT tracers (8 convolutions)
4D-Var Parameters: Decorrelation Scales

**Initial conditions correlations (m):**

- $\text{HdecayI(isFsur)} = 50.0d+3$  
  ! free-surface (16 convolutions)
- $\text{HdecayI(isUbar)} = 50.0d+3$  
  ! 2D U-momentum (16 convolutions)
- $\text{HdecayI(isVbar)} = 50.0d+3$  
  ! 2D V-momentum (16 convolutions)
- $\text{HdecayI(isUvel)} = 50.0d+3$  
  ! 3D U-momentum (16 convolutions)
- $\text{HdecayI(isVvel)} = 50.0d+3$  
  ! 3D V-momentum (16 convolutions)
- $\text{HdecayI(isTvar)} = 50.0d+3$  
  ! 1:NT tracers (16 convolutions)

- $\text{VdecayI(isUvel)} = 30.0d0$  
  ! 3D U-momentum (8 convolutions)
- $\text{VdecayI(isVvel)} = 30.0d0$  
  ! 3D V-momentum (8 convolutions)
- $\text{VdecayI(isTvar)} = 30.0d0$  
  ! 1:NT tracers (8 convolutions)

**Surface forcing correlations (m):**

- $\text{HdecayF(isUstr)} = 100.0d+3$  
  ! surface U-momentum stress (66 convolutions)
- $\text{HdecayF(isVstr)} = 100.0d+3$  
  ! surface V-momentum stress (66 convolutions)
- $\text{HdecayF(isTsur)} = 100.0d+3$  
  ! 1:NT surface tracer flux (66 convolutions)
# 4D-Var Parameters: Decorrelation Scales

**Open boundary conditions correlations (m):**

<table>
<thead>
<tr>
<th></th>
<th>1: west</th>
<th>2: south</th>
<th>3: east</th>
<th>4: north</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{\text{decayB}}(\text{isFsur}) )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
</tr>
<tr>
<td>( H_{\text{decayB}}(\text{isUbar}) )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
</tr>
<tr>
<td>( H_{\text{decayB}}(\text{isVbar}) )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
</tr>
<tr>
<td>( H_{\text{decayB}}(\text{isUvel}) )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
</tr>
<tr>
<td>( H_{\text{decayB}}(\text{isVvel}) )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
<td>( 100.0d+3 )</td>
</tr>
<tr>
<td>( H_{\text{decayB}}(\text{isTvar}) )</td>
<td>( 4\times100.0d+3 )</td>
<td>( 4\times100.0d+3 )</td>
<td>( \text{1:NT tracers} )</td>
<td></td>
</tr>
</tbody>
</table>

**Boundary edges to adjust (logical switches):**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{obc}}(\text{isFsur}) )</td>
<td>( \text{T} )</td>
<td>( \text{T} )</td>
<td>( \text{F} )</td>
<td>( \text{T} )</td>
</tr>
<tr>
<td>( L_{\text{obc}}(\text{isUbar}) )</td>
<td>( \text{T} )</td>
<td>( \text{T} )</td>
<td>( \text{F} )</td>
<td>( \text{T} )</td>
</tr>
<tr>
<td>( L_{\text{obc}}(\text{isVbar}) )</td>
<td>( \text{T} )</td>
<td>( \text{T} )</td>
<td>( \text{F} )</td>
<td>( \text{T} )</td>
</tr>
<tr>
<td>( L_{\text{obc}}(\text{isUvel}) )</td>
<td>( \text{T} )</td>
<td>( \text{T} )</td>
<td>( \text{F} )</td>
<td>( \text{T} )</td>
</tr>
<tr>
<td>( L_{\text{obc}}(\text{isVvel}) )</td>
<td>( \text{T} )</td>
<td>( \text{T} )</td>
<td>( \text{F} )</td>
<td>( \text{T} )</td>
</tr>
<tr>
<td>( L_{\text{obc}}(\text{isTvar}) )</td>
<td>( \text{T} )</td>
<td>( \text{T} )</td>
<td>( \text{F} )</td>
<td>( \text{T} )</td>
</tr>
</tbody>
</table>
Normalization Parameters File: c4dvar.in

4DVar assimilation input parameters.

Input parameters can be entered in ANY order, provided that the parameter KEYWORD (usually, upper case) is typed correctly followed by "=" or "==" symbols. Any comment lines are allowed and must begin with an exclamation mark (!) in column one. Comments may appear to the right of a parameter specification to improve documentation. Comments will be ignored during reading. Blank lines are also allowed and ignored. Continuation lines in a parameter specification are allowed and must be preceded by a backslash (\). In some instances, more than one value is required for a parameter. If fewer values are provided, the last value is assigned for the entire parameter array. The multiplication symbol (*), without blank spaces in between, is allowed for a parameter specification. For example, in a two grids nested application:

AKT_BAK == 2*1.0d-6 2*5.0d-6 ! m2/s

indicates that the first two entries of array AKT_BAK, in fortran column-major order, will have the same value of "1.0d-6" for grid 1, whereas the next two entries will have the same value of "5.0d-6" for grid 2.

In multiple levels of nesting and/or multiple connected domains step-ups, "Ngrids" entries are expected for some of these parameters. In such case, ...
Job Script: job_normalization.csh

1. Set path definition to one directory up in the tree.

   ```
   set Dir = `dirname ${PWD}`
   ```

2. Set string manipulations perl script.

   ```
   set SUBSTITUTE = ${ROMS_ROOT}/ROMS/Bin/substitute
   ```

3. Set model error, initial conditions, boundary conditions and surface forcing error covariance standard deviations files.

   ```
   set STDnameM = ../Data/wc13_std_m.nc
   set STDnameI = ../Data/wc13_std_i.nc
   set STDnameB = ../Data/wc13_std_b.nc
   set STDnameF = ../Data/wc13_std_f.nc
   ```

4. Set model error, initial conditions, boundary conditions and surface forcing error covariance normalization factors files.

   ```
   set NRMnameM = ../Data/wc13_nrm_m.nc
   set NRMnameI = ../Data/wc13_nrm_i.nc
   set NRMnameB = ../Data/wc13_nrm_b.nc
   set NRMnameF = ../Data/wc13_nrm_f.nc
   ```

5. Modify 4D-Var template input script and specify above files.

   ```
   set NORM = c4dvar.in
   if (-e $NORM) then
      /bin/rm $NORM
   endif
   cp s4dvar.in $NORM
   ```

   ```
   $SUBSTITUTE $NORM roms_std_m.nc $STDnameM
   $SUBSTITUTE $NORM roms_std_i.nc $STDnameI
   $SUBSTITUTE $NORM roms_std_b.nc $STDnameB
   $SUBSTITUTE $NORM roms_std_f.nc $STDnameF
   $SUBSTITUTE $NORM roms_nrm_m.nc $NRMnameM
   $SUBSTITUTE $NORM roms_nrm_i.nc $NRMnameI
   $SUBSTITUTE $NORM roms_nrm_b.nc $NRMnameB
   $SUBSTITUTE $NORM roms_nrm_f.nc $NRMnameF
   $SUBSTITUTE $NORM roms_obs.nc $OBSname
   $SUBSTITUTE $NORM roms_hss.nc wc13_hss.nc
   $SUBSTITUTE $NORM roms_lcz.nc wc13_lcz.nc
   $SUBSTITUTE $NORM roms_mod.nc wc13_mod.nc
   $SUBSTITUTE $NORM roms_err.nc wc13_err.nc
   ```
#!/bin/csh -f
#
# svn $Id: job_normalization.csh 977 2019-07-26 06:01:07Z arango $
# Copyright (c) 2002-2019 The ROMS/TOMS Group
# Licensed under a MIT/X style license
# See License_ROMS.txt
# 4D-Var error covariance normalization coefficients job script:
# This script NEEDS to be run before any run:
#
# (1) It copies a new clean nonlinear model initial conditions
# file. The nonlinear model is initialized from the
# background or reference state.
# (2) Specify model, initial conditions, boundary conditions, and
# surface forcing error covariance input standard deviations
# files.
# (3) Specify model, initial conditions, boundary conditions, and
# surface forcing error covariance input/output normalization
# factors filenames.
# (4) Create 4D-Var input script "c4dvar.in" from a template and
# specify the error covariance standard deviation, and error
# covariance normalization factors files to be used.
#
# Set path definition to one directory up in the tree.
Compile: build_roms.csh

1. Set a local environmental variable to define the path to the directories where all this project's files are kept.
   
   ```
   setenv MY_ROOT_DIR $(HOME)/ocean/toms/repository
   setenv MY_PROJECT_DIR $(PWD)
   ```

2. Location of your ROMS source code.
   
   ```
   setenv MY_ROMS_SRC $(MY_ROOT_DIR)/trunk
   ```

3. Path of Makefile configuration (*.mk) files
   
   ```
   setenv COMPILERS $(MY_ROMS_SRC)/Compilers
   setenv COMPILERS $(HOME)/Compilers/ROMS
   ```

4. Build script invoked CPP options.
   
   ```
   setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DNORMALIZATION"
   setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DANA_initial"
   setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DANA_SPONGE"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DADJUST_BOUNDARY"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DADJUST_WSTRESS"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DADJUST_STFLUX"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DBALANCE_OPERATOR"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DZETA_ELLIPTIC"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DCOLLECT_ALLREDUCE"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DREDUCE_ALLGATHER"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DDEBUGGING"
   #setenv MY_CPP_FLAGS "$\{MY_CPP_FLAGS\} -DPOSITIVE_ZERO"
   ```

5. Compiler selection environment variables.
   
   ```
   setenv USE_MPI on
   setenv USE_MPIF90 on
   setenv which_MPI openmp
   setenv FORT ifort
   ```

6. Use custom library paths
   
   ```
   #setenv USE_MY_LIBS no # use system default library paths
   setenv USE_MY_LIBS yes # use my customized library paths
   ```

   ```
   set MY_PATHS = $(COMPILERS)/my_build_paths.csh
   if (USE_MY_LIBS == 'yes') then
     source $(MY_PATHS) $(MY_PATHS)
   endif
   ```
#!/bin/csh -f
#
# svn $Id: build_roms.csh 977 2019-07-26 06:01:07Z arango $
#::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
# Copyright (c) 2002-2019 The ROMS/TOMS Group                           :::
#   Licensed under a MIT/X style license                                :::
#   See License_ROMS.txt                                                :::
#::::::::::::::::::::::::::::::::::::::::::::::::::::: Hernan G. Arango :::
#
# ROMS/TOMS Compiling CSH Script                                        :::
#
# Script to compile an user application where the application-specific :::
# files are kept separate from the ROMS source code.                    :::
#
# Q: How/why does this script work?                                     :::
#                                                                       :
# A: The ROMS makefile configures user-defined options with a set of    :::
#    flags such as ROMS_APPLICATION. Browse the makefile to see these.  :::
#    If an option in the makefile uses the syntax ?= in setting the      :::
#    default, this means that make will check whether an environment    :::
#    variable by that name is set in the shell that calls make. If so   :::
#    the environment variable value overrides the default (and the      :::
#    user need not maintain separate makefiles, or frequently edit      :::
#    the makefile, to run separate applications).                       :::
#
# Usage:                                                                :::
#                                                                       :
#    ./build_roms.csh [options]                                          :::
#                                                                       :
# Options:                                                              :::
#                                                                       :]
# svn $Id$
# # ROMS/TOMS Customized Compiling Libraries Script
# # This C-shell script sets the customized library paths needed by the
# # build script when the enviormental variable USE_MY_LIBS has a 'yes'
# # value.
# # For example, in build_roms.csh we have:
# #
# #       if ($USE_MY_LIBS == 'yes') then
# #          source ${COMPILERS}/my_build_paths.csh
# #       endif
# #
# set separator = `perl -e "print ':' x 100;"`

echo ""
echo "${separator}"
echo "Using customized library paths from: $1"
echo "${separator}"
Error Covariance Normalization Coefficients

**EXACT**

Free-surface  $T_x$  $T_y$  Net Heat Flux

**RANDOMIZATION**

Free-surface  $T_x$  $T_y$  Net Heat Flux
Error Covariance Normalization Coefficients

EXACT

3D U-momentum
3D V-momentum
Temperature
Salinity

RANDOMIZATION

3D U-momentum
3D V-momentum
Temperature
Salinity
References