EXERCISE 6: Analysis Cycle Observation Sensitivity

Introduction

During Lecture 5, we also showed how the sensitivity of scalar functions of the circulation to variations in the observations and observation array can be computed using the adjoint of 4D-Var. Specifically, the *posterior* vector of control increments can be written as:

$$\mathbf{z_a} = \mathbf{z_b} + K(\mathbf{d})$$

where $\mathbf{d} = \mathbf{y} - H\left(\mathbf{z_b}(t)\right)$ is the innovation vector, and $K(\mathbf{d})$ denotes the 4D-Var procedure which is a nonlinear function of **d** by virtue of the nature of the conjugate gradient algorithm. Any change $\delta\mathbf{y}$ in the observations leads to a first order change $\delta\mathbf{z_a} \approx \left(\partial K/\partial\mathbf{y}\right)\delta\mathbf{y}$ in the *posterior* control vector. Considering again the time average transport across 37N over the upper 500 m, I_{37N}, as an example of linear scalar function, the change in transport ΔI due to perturbations $\delta\mathbf{y}$ in the observations is given by:

$$\Delta I \approx \delta\mathbf{y}^{\mathrm{T}}\left(\partial K/\partial\mathbf{y}\right)^{\mathrm{T}}\sum_{i=1}^{N}\left(\mathbf{M_b^{\mathrm{T}}}\right)_i\mathbf{h}$$

where $\sum_{i=1}^{N}\left(\mathbf{M_b^{\mathrm{T}}}\right)_i\mathbf{h}$ represents ADROMS forced by **h**, and $\left(\partial K/\partial\mathbf{y}\right)^{\mathrm{T}}$ represents the adjoint of the linearized 4D-Var procedure, denoted $(\text{4D-Var})^{\mathrm{T}}$. Once a 4D-Var cycle has been performed, ΔI can be computed by running $(\text{4D-Var})^{\mathrm{T}}$ which involves running the each of the linearized 4D-Var inner-loops again, but in reverse order, combined with the adjoint of the Lanczos algorithm.

Running the observation impact driver

To compute the sensitivity of I_{37N} to changes in the observations, you must first perform a 4D-Var data assimilation calculation using RBL4D-Var.

Then go first to the directory **WC13/RBL4DVAR_analysis_sensitivity** and follow the directions in the **Readme** file. Also, be sure to change **Ninner** and **NHIS** in **roms_wc13_2hours.in** to be the same as you used for Exercise 3. Note that $(\text{4D-Var})^{\mathrm{T}}$ requires the same computational effort as 4D-Var.

Create a new subdirectory **EX6**, and save the solution in it for analysis and plotting to avoid overwriting solutions when playing with different CPP options and rerunning and recompiling:

```
mkdir EX6
mv Build_roms rbl4dvar.in *.nc log EX6
cp -p romsM roms_wc13_2hours.in EX6
```

where log is the ROMS standard output specified.

Plotting your results

To plot the results of your observation sensitivity calculation, use the Matlab script **plot_rbl4dvar_analysis_sensitivity.m**. You will need to edit the pathnames to point to **RBL4DVAR/EX3** as appropriate.

The plots that are generated correspond to the case where $\delta\mathbf{y}=\mathbf{d}$. In practice, $K(\mathbf{d})$ is a weakly nonlinear function of **d**, so in this case $K(\mathbf{d}+\delta\mathbf{y})=K(2\mathbf{d})\approx 2K(\mathbf{d})$, so that:

$$\Delta I \approx \mathbf{d}^{\mathrm{T}}\left(\partial K/\partial\mathbf{y}\right)^{\mathrm{T}}\sum_{i=1}^{N}\left(\mathbf{M}_{\mathbf{b}}^{\mathrm{T}}\right)_i\mathbf{h}\approx I\left(\mathbf{x}_{\mathbf{a}}\right)-I\left(\mathbf{x}_{\mathbf{b}}\right)$$

where $I\left(\mathbf{x}_{\mathbf{a}}\right)-I\left(\mathbf{x}_{\mathbf{b}}\right)$ is the transport increment of Exercise 5. Therefore, for $\delta\mathbf{y}=\mathbf{d}$, we expect ΔI to the same as that of Exercise 5 which you can confirm by comparing the plots of this exercise with those of Exercise 5. However, the contributions of the individual observation platforms to ΔI and $I\left(\mathbf{x}_{\mathbf{a}}\right)-I\left(\mathbf{x}_{\mathbf{b}}\right)$ will not be the same (there is no *a priori* reason why they should be), as you can also confirm. In the present case, ΔI is the change that would occur in I_{37N} if each observation y_i was changed by an amount equal to the corresponding increment d_i. Conversely, if $\delta\mathbf{y}=-\mathbf{d}$ instead, this is equivalent to changing each observation y_i by an amount $-d_i$. The effect of this in 4D-Var would be to yield a new innovation vector with all elements being zero, corresponding to perfect agreement between the model and the observations. If instead of considering $\delta\mathbf{y}=-\mathbf{d}$ we consider $\delta y_i=-d_i$ for only some of the observations (e.g. SSH) this would be equivalent to imposing perfect agreement between these observations and the model. In Lecture 5 we showed that this is a very efficient way of performing observation simulation experiments *without* rerunning the 4D-Var cycle with observations withheld. Therefore, if you *reverse* the sign of the colored bars in the plots for this exercise you can predict what the change in transport will be if observations from each platform are withheld separately or in combination.